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Local maximum-entropy approximation schemes: a seamless bridge between finite elements and meshfree methods. (English) Zbl 1146.74048

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Summary: We present a one-parameter family of approximation schemes, which we refer to as local maximum-entropy approximation schemes, that bridges continuously two important limits: Delaunay triangulation and maximum-entropy (max-ent) statistical inference. Local max-ent approximation schemes represent a compromise - in the sense of Pareto optimality - between the competing objectives of unbiased statistical inference from the nodal data and the definition of local shape functions of least width. Local max-ent approximation schemes are entirely defined by the node set and the domain of analysis, and the shape functions are positive, interpolate affine functions exactly, and have a weak Kronecker-delta property at the boundary. Local max-ent approximation may be regarded as a regularization, or thermalization, of Delaunay triangulation which effectively resolves the degenerate cases resulting from the lack or uniqueness of the triangulation. Local max-ent approximation schemes can be taken as a convenient basis for the numerical solution of PDEs in the style of meshfree Galerkin methods. In test cases characterized by smooth solutions we find that the accuracy of local max-ent approximation schemes is vastly superior to that of finite elements.

MSC:

74S05 Finite element methods applied to problems in solid mechanics

74S30 Other numerical methods in solid mechanics (MSC2010)

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[Delaunay triangulation](#); [statistical inference](#); [regularization](#)

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References:

- [1] Nayroles, *Computational Mechanics* 10 pp 307– (1992)
- [2] Belytschko, *International Journal for Numerical Methods in Engineering* 37 pp 229– (1994)
- [3] Liu, *Computer Methods in Applied Mechanics and Engineering* 143 pp 113– (1997)
- [4] , , . Chapter meshfree methods. *Encyclopedia of Computational Mechanics*, vol. 1. Wiley: Chichester, 2004; 279–309.
- [5] Duarte, *Computer Methods in Applied Mechanics and Engineering* 139 pp 237– (1996)
- [6] Babuška, *International Journal for Numerical Methods in Engineering* 40 pp 727– (1997)
- [7] Jaynes, *Physical Review* 106 pp 620– (1957)
- [8] *Convex Analysis*. Princeton University Press: Princeton, NJ, 1970. · [Zbl 0932.90001](#) · [doi:10.1515/9781400873173](#)
- [9] , . *Bézier and B-spline Techniques*. Springer: Berlin, 2002. · [doi:10.1007/978-3-662-04919-8](#)
- [10] *The Approximation of Continuous Functions by Positive Linear Operators*. Springer: Berlin, 1972. · [Zbl 0276.41011](#) · [doi:10.1007/BFb0059493](#)
- [11] Sukumar, *International Journal for Numerical Methods in Engineering* 43 pp 839– (1998)
- [12] Cirak, *International Journal for Numerical Methods in Engineering* 47 pp 2039– (2000)
- [13] Cottin, *Journal of Inequalities and Applications* 4 pp 91– (1999)
- [14] Fernández-Méndez, *Computer Methods in Applied Mechanics and Engineering* 193 pp 1257– (2004)
- [15] Karlin, *Memoirs of the American Mathematical Society* 12 (1953)
- [16] Tagliani, *Applied Mathematics and Computation* 110 pp 105– (2000) · [Zbl 0987.60027](#)
- [17] *Mathematical Foundations of Information Theory*. Dover: New York, 1957. · [Zbl 0088.10404](#)
- [18] Shannon, *The Bell System Technical Journal* 27 pp 379– (1948) · [Zbl 1154.94303](#) · [doi:10.1002/j.1538-7305.1948.tb01338.x](#)
- [19] Mead, *Journal of Mathematical Physics* 25 pp 2404– (1984)
- [20] , , . Barycentric coordinates for convex sets, 2005, submitted.

- [21] Floater, *Advances in Computational Mathematics* (2005)
- [22] Sukumar, *International Journal for Numerical Methods in Engineering* 61 pp 2159– (2004)
- [23] Rajan, *Discrete and Computational Geometry* 12 pp 189– (1994)
- [24] . *Convex Optimization*. Cambridge University Press: Cambridge, U.K., 2004. · [Zbl 1058.90049](#) · [doi:10.1017/CBO9780511804441](#)
- [25] Attouch, *SIAM Journal on Optimization* 6 pp 769– (1996)
- [26] Attouch, *Nonlinear Analysis* 36 pp 373– (1999)
- [27] Wandzura, *Computers and Mathematics with Applications* 45 pp 1829– (2003)
- [28] Breitkopf, *Computer Methods in Applied Mechanics and Engineering* 193 pp 1203– (2004)
- [29] . *Theory of Elasticity*. McGraw-Hill: New York, 1951.
- [30] Organ, *Computational Mechanics* 18 pp 225– (1996)
- [31] Belytschko, *Computer Methods in Applied Mechanics and Engineering* 139 pp 3– (1996)
- [32] Krysl, *Computer Methods in Applied Mechanics and Engineering* 148 pp 257– (1997)

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