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**On an extended clarifier-thickener model with singular source and sink terms.** (English)

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Summary: A one-dimensional model of clarifier-thickener units in engineering applications can be expressed as a conservation law with a flux that is discontinuous with respect to the spatial variable. This model also includes a singular feed source. In this paper, the clarifier-thickener model studied in a previous paper [Numer. Math. 97, 25–65 (2004; Zbl 1053.76047)] is extended by a singular sink through which material is extracted from the unit. A difficulty is that in contrast to the singular source, the sink term cannot be incorporated into the flux function; rather, the sink is represented by a new non-conservative transport term. To focus on the new analytical difficulties arising due to this non-conservative term, a reduced problem is formulated, which contains the new sink term of the extended clarifier-thickener model, but not the source term and flux discontinuities. The paper is concerned with numerical methods for both models (extended and reduced) and with the well-posedness analysis for the reduced problem. For the reduced problem, a definition of entropy solutions, based on Kruškov-type entropy functions and fluxes, is provided. Jump conditions are derived and uniqueness of the entropy solution is shown. Existence of an entropy solution is shown by proving convergence of a monotone difference scheme. Two variants of the numerical scheme are introduced. Numerical examples illustrate that all three variants converge to the entropy solution, but introduce different amounts of numerical diffusion.

**MSC:**

- 35L60 First-order nonlinear hyperbolic equations
- 76M20 Finite difference methods applied to problems in fluid mechanics
- 76T20 Suspensions
- 65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs

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