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On the elimination of the sweeping interactions from theories of hydrodynamic turbulence.
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Summary: We revisit the claim that the Eulerian and quasi-Lagrangian time correlation tensors are equal. This statement allows us to transform the results of an MSR [*P. Martin, E. Siggia and H. Rose*, *Phys. Rev. A* 8, 423–437 (1973)] quasi-Lagrangian statistical theory of hydrodynamic turbulence back to the Eulerian representation. We define a hierarchy of homogeneity symmetries between incremental homogeneity and global homogeneity. It is shown that both the elimination of the sweeping interactions and the derivation of the 4/5-law require a homogeneity assumption stronger than incremental homogeneity but weaker than global homogeneity. The quasi-Lagrangian transformation, on the other hand, requires an even stronger homogeneity assumption which is many-time rather than one-time but still weaker than many-time global homogeneity. We argue that it is possible to relax this stronger assumption and still preserve the conclusions derived from theoretical work based on the quasi-Lagrangian transformation.

MSC:

76F05 Isotropic turbulence; homogeneous turbulence

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local homogeneity; quasi-Lagrangian statistical theory; global homogeneity

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