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Some rigorous results on a stochastic GOY model. (English) Zbl 1107.82057
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Summary: A stochastic infinite dimensional version of the Gledzer-Ohkitani-Ymada model is rigorously investigated. Well posedness of strong solutions, existence and p-integrability of invariant measures is proved. Existence of solutions to the zero viscosity equation is also proved. With these preliminary results, the asymptotic exponents ζ_p of the structure function are investigated. Necessary and sufficient conditions for $\zeta_2 \geq 2/3$ and $\zeta_2 = 2/3$ are given and discussed on the basis of numerical simulations.

MSC:

82C41 Dynamics of random walks, random surfaces, lattice animals, etc. in time-dependent statistical mechanics
76D05 Navier-Stokes equations for incompressible viscous fluids
76F05 Isotropic turbulence; homogeneous turbulence

Keywords:

stochastic GOY model; invariant measures; structure function; asymptotic exponents

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