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Solvability in the large for a class of vector fields on the torus. (English) Zbl 1157.35304
J. Math. Pures Appl. (9) 86, No. 5, 427-447 (2006).

Summary: We study a class of complex vector fields defined on the two-torus of the form $L = \partial/\partial t + (a(x, t) + ib(x, t))\partial/\partial x$, $a, b \in C^\infty(\mathbb{T}^2; \mathbb{R})$, $b \neq 0$. We view L as an operator acting on smooth functions and present conditions for L to have either a closed range or a finite-codimensional range. Our results involve, besides condition (\mathcal{P}) of Nirenberg and Treves, the behavior of $a + ib$ near each one-dimensional Sussmann orbit homotopic to the unit circle. One of the main goals of our work is to provide some clarification about the role played by the coefficient a in the validity of the above properties of the range.

MSC:

- 35A21 Singularity in context of PDEs
- 35F05 Linear first-order PDEs
- 47F05 General theory of partial differential operators (should also be assigned at least one other classification number in Section 47-XX)
- 35A05 General existence and uniqueness theorems (PDE) (MSC2000)

Cited in **13** Documents

Keywords:

closed range; finite-codimensional range; Sussmann orbit

Full Text: [DOI](#)

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