Bludov, V. V.; **Glass, A. M. W.**

On the variety generated by all nilpotent lattice-ordered groups. (English) Zbl 1110.06019

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The variety of weakly abelian lattice-ordered groups was introduced in 1974 by J. Martinez. It is defined by the identity: $x^{-1}(y \vee 1)x \vee (y \vee 1)^2 = (y \vee 1)^2$. The present paper deals with the variety generated by all nilpotent lattice-ordered groups. Its main results are the following:

Theorem A. There is a centre-by-metabelian weakly Abelian ordered group that does not belong to the variety of lattice-ordered groups generated by all nilpotent lattice-ordered groups. (Note that this result answers two question of V. M. Kopytov.)

Theorem B. The quasivariety generated by all nilpotent lattice-ordered groups is the same as the variety generated by all nilpotent lattice-ordered groups. (Note that the proof of Theorem B also gives a set of defining identities for this variety.)

Theorem C. Every abelian-by-nilpotent weakly abelian lattice-ordered group belongs to the variety of lattice-ordered groups generated by all nilpotent lattice-ordered groups.

All results of the paper are new and carefully proved. The presentation is clear, with many examples, and so the paper contributes to the development of this important research domain.

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MSC:

06F15 Ordered groups
20F18 Nilpotent groups
20F12 Commutator calculus

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nilpotent group; residually torsion-free-nilpotent; variety; quasi-variety; commutator calculus; lattice-ordered group; weakly abelian

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[16] The Black Swamp Problem Book is edited by W. Charles Holland (Bowling Green State University, Ohio 43403, U.S.A.) and is kept there by him (in the formerly Black Swamp region of Ohio).