Khrennikov, A. Yu.
Non-Archimedean analysis and its applications. (Неархимедов анализ и его приложения.)
(Russian) | Zbl 1104.46047 |

Moscow: Fiziko-Matematicheskaya Literatura (ISBN 5-9221-0191-9). 216 p. (2003).

The book is an introduction to the author's results in non-Archimedean analysis and non-Archimedean models of mathematical physics. It overlaps only slightly (basic notions given in Chapter 1) with other books devoted to non-Archimedean analysis and related branches of mathematics. Let us describe the contents of the book.

In Chapter 2, the author introduces distributions over K^n with values in a non-Archimedean field K or its extension. As the spaces of test functions, various spaces of entire K-valued functions are chosen. Then the author defines a kind of Laplace (or Fourier) transform which makes it possible to define some distributions prescribing their transforms. In particular, formulas resembling the classical ones lead to definitions of the Gauss and Feynman distributions. Their properties are quite different from those of their classical counterparts. For example, the Gaussian distribution is defined only on entire test functions and cannot be extended to continuous ones. In Chapter 3, analogs of some of the above constructions are given for functions on infinite-dimensional Banach spaces over K.

Chapter 4 is devoted to quantum mechanics with K-valued wave functions. Analogs of the Schrödinger, Heisenberg, and Liouville equations are introduced and studied. The author sketches possible applications to cosmology.

Dealing with p-adic-valued wave functions and wishing to give a probabilistic interpretation resembling that of conventional quantum mechanics, one cannot avoid introducing p-adic-valued probabilities. Such a probability theory has been developed by the author and is given in detail in Chapter 5. This includes an appropriate integration theory and results about some classes of random variables, including those similar to the Bernoulli ones. The latter are also applied to some biological models.

The notion of p-adic-valued probability is a natural extension of von Mises's frequency approach. This is discussed in Chapter 6, together with general considerations regarding p-adic statistical modeling, complexity, etc.

In Chapter 7, in terms of p-adic-valued probabilities, some aspects of the theory of p-adic-valued stochastic processes are considered (this should not be confused with the existing theory of such processes developed in the framework of the usual probability theory). In particular, a version of white noise theory is given.

Finally, Chapter 8 is devoted to recent attempts to use methods of p-adic analysis in order to create new algorithms of image processing. There are two types of such algorithms, one of which uses the ultrametric topology on the spectra of images, while the second employs the algebraic structure of the field of p-adic numbers.

An interested reader can find further related material in other books by the author [see, in particular, "Non-Archimedean analysis: quantum paradoxes, dynamical systems and biological models" (Math. Appl. 427, Kluwer Acad. Publ., Dordrecht) (1997; Zbl 0920.11087); "Interpretations of probability" (VSP, Utrecht) (1999; Zbl 1060.81003)].

Reviewer: Anatoly N. Kochubei (MR 2005c:46111)

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