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Counting unrooted maps on the plane. (English) Zbl 1112.05054
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A plane map is a 2-cell imbedding of a connected multigraph (loops and multiple edges allowed) on the sphere, with a distinguished face. The map is rooted by also distinguishing an oriented edge; an unrooted plane map is an equivalence class of plane maps under orientation-preserving homeomorphism. In a series of three papers [*Can. J. Math.* 35, 417–435 (1983; [Zbl 0519.05041](#)); *Discrete Math.* 282, No. 1–3, 209–221 (2004; [Zbl 1051.05049](#)) and *Eur. J. Comb.* 26, No. 5, 651–663 (2005; [Zbl 1070.05050](#))], the authors enumerated unrooted planar (no distinguished face) n -edge maps of various classes, including all maps, non-separable maps, Eulerian maps, and loopless maps. In the present paper, they employ the same technique to get closed formulae for counting unrooted plane maps of all these classes and their duals. In contrast to the rooted case, where the corresponding formulae are all sum-free, the formulae obtained for unrooted maps contain a sum over the divisors of n . Also counted are unrooted two-vertex plane maps.

Reviewer: [Arthur T. White \(Kalamazoo\)](#)

MSC:

[05C30](#) Enumeration in graph theory

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Keywords:

[rooted planar map](#); [unrooted plane map](#); [quotient map](#); [sum-free formula](#); [outside face](#)

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