Langa, José A.; **Łukaszewicz, G.**; **Real, J.**

Finite fractal dimension of pullback attractors for non-autonomous 2D Navier–Stokes equations in some unbounded domains. (English) Zbl 1113.37055

Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 66, No. 3, 735-749 (2007).

The authors analyze the long-term behavior of solutions of the non-autonomous two-dimensional Navier-Stokes equations on unbounded domains for which the Poincaré inequality holds true. They establish sufficient conditions which imply that the pullback attractor (a generalization of the concept of a global attractor in autonomous dynamical systems to the non-autonomous case; see *V. V. Chepyzhov* and *M. I. Vishik* [Attractors for equations of mathematical physics. Colloquium Publications. American Mathematical Society. 49. Providence, RI: American Mathematical Society (AMS) (2002; Zbl 0986.35001)]) of the associated evolution process has finite fractal dimension. The key assumption in this study is the boundedness of the non-autonomous terms in the past. This assumption makes the concept of pullback attractors applicable and allows to deduce "uniform pullback asymptotic compactness" (a weak form of compactness) for the evolution process. The compactness property replaces the lack of compact Sobolev embeddings due to unbounded domains.

Reviewer: Thomas Hagen (Memphis)

MSC:

37L30 Infinite-dimensional dissipative dynamical systems–attractors and their dimensions, Lyapunov exponents
35B41 Attractors
35Q30 Navier-Stokes equations

Cited in **18** Documents

Keywords:

long-term behavior; pullback attractor; asymptotic compactness

Full Text: DOI

References:

[1] Caraballo, T.; Langa, J.A.; Robinson, J.C., Stability and random attractors for a reaction – difussion equation with multiplicative noise, Discrete contin. dyn. syst., 6, 4, 875-892, (2000) · Zbl 1011.37031

[2] Caraballo, T.; Langa, J.A.; Valero, J., The dimension of attractors of nonautonomous partial differential equations, Anziam j., 45, 2, 207-222, (2003) · Zbl 1047.35024

[3] Caraballo, T.; Łukaszewicz, G.; Real, J., Pullback attractors for asymptotically compact nonautonomous dynamical systems, Nonlinear anal., 64, 3, 484-498, (2006) · Zbl 1128.37019

[4] Chepyzov, V.V.; Vishik, M.I., Attractors for equations of mathematical physics, () · Zbl 0986.35001

[5] Constantin, P.; Foias, C.; Temam, R., Attractors representing turbulent flows, Mem. amer. math. soc., 53, (1984) · Zbl 0567.35070

[6] Crauel, H.; Debussche, A.; Flandoli, F., Random attractors, J. dynam. differential equations, 9, 2, 307-341, (1995) · Zbl 0884.58064

[7] Crauel, H.; Flandoli, F., Additive noise destroys a pitchfork bifurcation, J. dynam. differential equations, 10, 259-274, (1998) · Zbl 0907.34042

[8] Eden, A.; Foias, C.; Nicolaenko, B.; Temam, R., Exponential attractors for dissipative evolution equations, () · Zbl 0842.58056

[9] Kloeden, P.E., A Lyapunov function for pullback attractors of nonautonmous differential equations, Electron. J. differ. equ. conf., 05, 91-102, (2000) · Zbl 0964.34041

[10] Kloeden, P.E., Pullback attractors in nonautonomous difference equations, J. differ. equations appl., 6, 1, 33-52, (2000) · Zbl 0961.39007

[11] Kloeden, P.E.; Schmalfuss, B., Asymptotic behaviour of non-autonomous difference inclusions, Systems and control lett., 33, 275-280, (1998) · Zbl 0902.93043

[12] Haraux, A., ()

[13] Hou, Y.; Li, K., The uniform attractor for the 2D non-autonomous navier – stokes flow in some unbounded domains, Nonlinear

anal., 58, 5-6, 609-630, (2004) · Zbl 1057.35031

[14] Ilyin, A.A., Lieb – thirring inequalities on the N-sphere and in the plane and some applications, Proc. London math. soc. (3), 67, 159-182, (1993) · Zbl 0789.58079

[15] Langa, J.A.; Robinson, J.C.; Suárez, A., Forwards and pullback behaviour of a non-autonomous lotka – volterra system, Nonlinearity, 16, 1-17, (2003) · Zbl 1044.34009

[16] Langa, J.A.; Robinson, J.C.; Suárez, A., Bifurcation from zero of a complete trajectory for non-autonomous logistic pdes, Internat. J. bifur. chaos, 15, 8, 2663-2669, (2005) · Zbl 1092.35514

[17] Langa, J.A.; Schmalfuss, B., Finite dimensionality of attractors for non-autonomous dynamical systems given by partial differential equations, Stoch. dyn., 4, 3, 385-404, (2004) · Zbl 1057.37069

[18] Lions, J.L., Quelques méthodes de résolution des problèmes aux limites non linéaires, (1969), Dunod, Gauthier-Villars Paris · Zbl 0189.40603

[19] Łukaszewicz, G.; Sadowski, W., Uniform attractor for 2D magneto-micropolar fluid flow in some unbounded domains, Z. angew. math. phys., 55, 1-11, (2004)

[20] Moise, I.; Rosa, R.; Wang, X., Attractors for noncompact nonautonomous systems via energy equations, Discrete contin. dyn. syst., 10, 1-2, 473-496, (2004) · Zbl 1060.35023

[21] Robinson, J.C., Infinite dimensional dynamical systems, (2001), Cambridge University Press · Zbl 1026.37500

[22] Rosa, R., The global attractor for the 2D navier – stokes flow on some unbounded domains, Nonlinear anal., 32, 1, 71-85, (1998) · Zbl 0901.35070

[23] Schmalfuss, B., Attractors for non-autonomous dynamical systems, (), 684-689 · Zbl 0971.37038

[24] Sell, G.R., Non-autonomous differential equations and topological dynamics, I, II, Trans. amer. math. soc., 127, 241-262, (1967), 263-283 · Zbl 0189.39602

[25] Temam, R., Navier – stokes equations, theory and numerical analysis, (1979), North Holland Amsterdam · Zbl 0426.35003

[26] Temam, R., Infinite dimensional dynamical systems in mechanics and physics, (1988), Springer-Verlag New York · Zbl 0662.35001