Cordaro, Giuseppe

Multiple solutions to a perturbed Neumann problem. (English) Zbl 1387.35154

Stud. Math. 178, No. 2, 167-175 (2007).

Summary: We consider the perturbed Neumann problem

$$\begin{cases} -\Delta u+\alpha(x)u=\alpha(x)f(u)+\lambda g(x,u) & \text{a.e.in } \Omega, \\ \partial u/\partial\nu=0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is an open bounded set in $\mathbb{R}^N$ with boundary of class C^2, $\alpha\in L^\infty(\Omega)$ with $\operatorname{ess\,inf}_\Omega\alpha>0$, $f:\mathbb{R}\to\mathbb{R}$ is a continuous function and $g:\Omega\times\mathbb{R}\to\mathbb{R}$, besides being a Carathéodory function, is such that, for some $p>N$, $\sup_{|s|\le t}|g(\cdot,s)|\in L^p(\Omega)$ and $g(\cdot,t)\in L^\infty(\Omega)$ for all $t\in\mathbb{R}$. In this setting, supposing only that the set of global minima of the function $\frac{1}{2}\xi^2-\int_0^\xi f(t)\,dt$ has $M\ge 2$ bounded connected components, we prove that, for all $\lambda\in\mathbb{R}$ small enough, the above Neumann problem has at least $M+$ integer part of $M/2$ distinct strong solutions in $W^{2,p}(\Omega)$.

MSC:

35J20 Variational methods for second-order elliptic equations

35J60 Nonlinear elliptic equations

35J25 Boundary value problems for second-order elliptic equations

47J30 Variational methods involving nonlinear operators

58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces

Keywords:

Neumann problem; connected component; multiplicity of solutions; weak solution; strong solution

Full Text: DOI