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Non-planar fronts in Boussinesq reactive flows. (English) Zbl 1157.35469

Ann. Inst. Henri Poincaré, Anal. Non Linéaire 23, No. 4, 407-437 (2006).

Summary: We consider the reactive Boussinesq equations in a slanted cylinder, with zero stress boundary conditions and arbitrary Rayleigh number. We show that the equations have non-planar traveling front solutions that propagate at a constant speed. We also establish uniform upper bounds on the burning rate and the flow velocity for general front-like initial data for the Cauchy problem.

MSC:

35Q53 KdV equations (Korteweg-de Vries equations)
76D99 Incompressible viscous fluids
76V05 Reaction effects in flows

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