

Garbaczewski, Piotr

Differential entropy and dynamics of uncertainty. (English) Zbl 1124.82014
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The author uses the notion of differential entropy for a continuous probability density in order to study time-dependent problems in both classical and quantum case. The paper begins introducing concepts related to the Shannon entropy and its relation to information and uncertainty. A connection is then set between Shannon entropy of a discrete probability measure and differential entropy of a related continuous probability density, which contrary to the von Neumann entropy does not vanish on pure states, quantifying the degree of probability localization or delocalization. The localization level of probability densities is then analyzed by means of entropy powers and Fisher information measure. Time dependent problems both in dissipative and quantum mechanical cases are dealt with, studying dynamics of differential entropy and of the Fisher information functional.

Reviewer: [Bassano Vacchini \(Milano\)](#)

MSC:

82C31 Stochastic methods (Fokker-Planck, Langevin, etc.) applied to problems in time-dependent statistical mechanics
94A17 Measures of information, entropy

Cited in **10** Documents

Keywords:

[entropy functionals](#); [Shannon entropy](#); [Fisher information](#); [Kullback-Leibler entropy](#); [entropy dynamics](#); [Smoluchowski process](#); [information](#); [localization](#); [uncertainty](#)

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