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Existence and nonexistence of traveling waves and reaction-diffusion front propagation in periodic media. (English) Zbl 1102.35340

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Summary: We review the existence results of traveling wave solutions to the reaction-diffusion equations with periodic diffusion (convection) coefficients and combustion (bistable) nonlinearities. We prove that whenever traveling waves exist, the solutions of the initial value problem with either frontlike or pulselike data propagate with the constant effective speeds of traveling waves in all suitable directions. In the case of bistable nonlinearity and one space dimension, we give an example of nonexistence of traveling waves which causes quenching (localization) of wavefront propagation. Quenching (localization) only occurs when the variations of the media from their constant mean values are large enough. Our related numerical results also provide evidence for this phenomenon in the parameter regimes not covered by the analytical example. Finally, we comment on the role of the effective wave speeds in determining the effective wavefront equation (Hamilton-Jacobi equation) of the reaction-diffusion equations under the small-diffusion, fast-reaction limit with a formal geometric optics expansion.

MSC:

35K57 Reaction-diffusion equations

35B40 Asymptotic behavior of solutions to PDEs

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Keywords:

Reaction-diffusion equations; homogenization; traveling waves; maximum principle; Hamilton-Jacobi equations

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