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MUSTA fluxes for systems of conservation laws. (English) Zbl 1097.65091
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Summary: Numerical fluxes for hyperbolic systems and we first present a numerical flux, called GFORCE, that is a weighted average of the Lax-Friedrichs and Lax-Wendroff fluxes. For the linear advection equation with constant coefficient, the new flux reduces identically to that of the Godunov first-order upwind method.

Then we incorporate GFORCE in the framework of the MUSTA approach [cf. *E. F. Toro*, Multi-Stage Predictor-Corrector Fluxes for Hyperbolic Equations. Technical Report NI03037-NPA, Isaac Newton Institute for Mathematical Sciences, University of Cambridge, UK, 17th June, 2003], resulting in a version that we call GMUSTA. For nonlinear systems this gives results that are comparable to those of the Godunov method in conjunction with the exact Riemann solver or complete approximate Riemann solvers, noting however that in our approach, the solution of the Riemann problem in the conventional sense is avoided. Both the GFORCE and GMUSTA fluxes are extended to multi-dimensional nonlinear systems in a straightforward unsplit manner, resulting in linearly stable schemes that have the same stability regions as the straightforward multi-dimensional extension of Godunov's method.

The methods are applicable to general meshes. The schemes of this paper share with the family of centred methods the common properties of being simple and applicable to a large class of hyperbolic systems, but the schemes of this paper are distinctly more accurate. Finally, we proceed to the practical implementation of our numerical fluxes in the framework of high-order finite volume WENO methods for multi-dimensional nonlinear hyperbolic systems. Numerical results are presented for the Euler equations and for the equations of magnetohydrodynamics.

MSC:

- 65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
- 35L65 Hyperbolic conservation laws
- 65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs
- 35L45 Initial value problems for first-order hyperbolic systems
- 76N15 Gas dynamics, general
- 76W05 Magnetohydrodynamics and electrohydrodynamics
- 76M12 Finite volume methods applied to problems in fluid mechanics

Cited in **2** Reviews
Cited in **22** Documents

Keywords:

hyperbolic conservation laws; upwind methods; GFORCE flux; MUSTA fluxes; WENO methods; Euler equations; MHD equations; Lax-Friedrich method; nLax-Wendroff method; Godunov method; linear advection equation; Riemann solver; stability; finite volume; numerical results; magnetohydrodynamics

Software:

GFORCE; HE-E1GODF; HLLE

Full Text: [DOI](#)

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