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The attractors for the nonhomogeneous nonautonomous Navier–Stokes equations. (English)

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The authors consider the inhomogeneous Navier-Stokes equations

$$\begin{aligned} &u_t - \nu\Delta u + (u\nabla)u + \nabla p = f \\ &\operatorname{div} u = 0 \text{ on } \Omega, \quad u = \varphi \text{ on } \partial\Omega, \quad \Omega \subseteq \mathbb{R}^2 \end{aligned} \tag{1}$$

on a bounded Lipschitz domain Ω in $\mathbb{R}^2$. One assumes

$$f = f(x,t) \in \mathcal{L}^2_{\text{loc}}((0,T),E), \quad \varphi \in \mathcal{L}^2(\partial\Omega) \tag{2}$$

where $E = \operatorname{dom}(A^{\frac{1}{4}})$, with $A = -P_\Delta$ the Stokes operator associated with (1). The aim is to prove the existence of a global attractor for (1). To do so, the authors need several preparatory steps. First, using a suitable background flow ψ, eq. (1) is transformed into a new one, based on Dirichlet boundary conditions, i.e.:

$$v_t + \nu Av + B(v,v) + B(v,\psi) + B(\psi,v) = P(f+\nu F) - B(\psi,\psi) \tag{3}$$

where F is an additional force term induced by the background flow ψ. In order to prove the existence of an attractor for (3), the authors have to rely on work of *V. V. Chepyzhov* and *M. I. Vishik* [Am. Math. Soc. Colloq. Publ. 49, 363 p. (2002; Zbl 0986.35001)]; they introduce a number of notions and discuss their properties. Thus one has the notion of indexed process $\{U_\sigma(t,\tau) \mid t \geq \tau,\ \tau \in \mathbb{R}, \sigma \in \Sigma\}$ where Σ is the index space (a metric space), σ the symbol of the process and $\{U_\sigma(t,\tau)\}$ a family of mappings on a Banach space E such that

$$U(t,s)U(s,\tau) = U(t,\tau), \quad U(\tau,\tau) = \mathrm{Id}, \quad t \geq s \geq \tau, \quad \tau \in \mathbb{R}.$$

In terms of this notion, the relevant topological concepts such as absorbing set, ω-limit set, uniform attractor etc. are introduced, and some of their properties summarized. Criteria (Thms. 4.1, 4.2) for the existence of a uniform attractor are given. Finally, the index space is made precise: it is based on the translates $(T_h f)(s) = f(h+s)$ induced by the exterior force f in (1) resp. (3). In the main section 6 the existence of a uniform attractor in the sense of Chepyzhov and Vishik (loc. cit.) is proved. First, it is noted that existence of global solutions of (3) is guaranteed by a Galerkin method; for details the reader is referred to *R. M. Brown, P. A. Perry*, and *Zh. Shen* [Indiana Univ. Math. J. 49, 81–112 (2000; Zbl 0969.35105)] where a proof in a comparable situation is given. Then one proceeds to the proof of the main Theorem 6.1 which asserts the existence of a uniform attractor for (3). The proof involves lengthy estimates, based in part on the paper of Brown, Perry, Shen (loc. cit.). Theorem 6.2 finally asserts that if $f(x,s)$ is translation compact in $D(A^{-\frac{1}{4}})$ then the attractor in question is compact.

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MSC:

35Q30 Navier-Stokes equations
37L30 Infinite-dimensional dissipative dynamical systems–attractors and their dimensions, Lyapunov exponents
76D05 Navier-Stokes equations for incompressible viscous fluids

Keywords:

measure of noncompactness; inhomogeneous Navier-Stokes equations; bounded Lipschitz domain; Stokes

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