Arriola, L. M.; Beyer, W. A.
Stability of the Cauchy functional equation over p-adic fields. (English) Zbl 1099.39019
Real Anal. Exch. 31(2005-2006), No. 1, 125-132 (2006).

During the last three decades the p-adic number field $\mathbb{Q}_p$ has gained the interest of physicists for their research in particular in problems coming from quantum physics, p-adic strings and superstrings [cf. *A. Khrennikov*, Non-archimedean analysis: quantum paradoxes, dynamical systems and biological models. Mathematics and its Applications (Dordrecht). 427. Dordrecht: Kluwer Academic Publishers. (1997; Zbl 0920.11087)]. A key property of p-adic numbers is that they do not satisfy the Archimedean axiom: for all $x, y > 0$, there exists an integer n such that $x < ny$.

The authors investigate the stability of approximate additive mappings $f : \mathbb{Q}_p \to \mathbb{R}$. They show that if $f : \mathbb{Q}_p \to \mathbb{R}$ is a continuous mapping for which there exists a fixed ε such that $|f(x+y) - f(x) - f(y)| \leq \varepsilon$ $(x, y \in \mathbb{Q}_p)$, then there exists a unique additive mapping $T : \mathbb{Q}_p \to \mathbb{R}$ such that $|f(x) - T(x)| \leq \varepsilon$ for all $x \in \mathbb{Q}_p$. It seems that they do not use any essential property of p-adic numbers in their proofs.

Reviewer: Mohammad Sal Moslehian (Mashhad)

MSC:

39B82 Stability, separation, extension, and related topics for functional equations
46S10 Functional analysis over fields other than $\mathbb{R}$ or $\mathbb{C}$ or the quaternions; non-Archimedean functional analysis
11S80 Other analytic theory (analogues of beta and gamma functions, p-adic integration, etc.)
39B22 Functional equations for real functions

Keywords:
stability; additive mapping; continuity; p-adic numbers

Full Text: DOI