Le Calvez, Patrice

Periodic orbits of Hamiltonian homeomorphisms of surfaces. (English) Zbl 1101.37031

Duke Math. J. 133, No. 1, 125-184 (2006).

J. Franks and *M. Handel* [Geom. Topol. 7, 713–756 (2003; Zbl 1034.37028)] proved in 2003 that any nontrivial Hamiltonian diffeomorphisms of a closed surface whose genus is at least one has periodic orbits of arbitrarily large period. Moreover, they established a related result for nontrivial area-preserving diffeomorphisms of the sphere with at least three fixed-points. The author of this paper extends these results to the case of homeomorphisms. The most general result is the following: Let M be an orientable surface (not necessarily compact) that is not simply connected, and let F be a homeomorphism of M with no wandering point. If F admits a lift $\widetilde{F}$ to the universal covering spaces $\widetilde{M}$ that commutes with the covering transformations, and if there is a closed disk D such that the Lefschetz index $i(\widetilde{F}, D) \geq 1$, then $\widetilde{F}$ has periodic points of arbitrarily large period. For a sphere, the result is: If F is a nontrivial orientation-preserving homeomorphism of a sphere S with at least three fixed-points, and if F has no wandering point, then there exist periodic points with arbitrarily large periods. The author's proof makes use of an equivariant foliated version of Brouwer's plane translation theorem, as well as properties of the linking number of fixed-points. The author concludes with a discussion of related problems and open questions.

Reviewer: William J. Satzer jun. (St. Paul)

MSC:

37E30	Dynamical systems involving homeomorphisms and diffeomorphisms of planes and surfaces	Cited in **23** Documents
37E45	Rotation numbers and vectors	
37J10	Symplectic mappings, fixed points (dynamical systems) (MSC2010)	
37B25	Stability of topological dynamical systems	
37E35	Flows on surfaces	

Keywords:

periodic orbits of homeomorphisms; equivariant foliation; plane translation theorem; Hamiltonian systems; rotation vector

Full Text: DOI

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