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Dual presentations of braid groups of affine type \tilde{A} . (Présentations duales des groupes de tresses de type affine \tilde{A} .) (French) [Zbl 1143.20020](#)

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Summary: Artin-Tits groups of spherical type have two well-known Garside structures, coming respectively from the divisibility properties of the classical Artin monoid and of the dual monoid. For general Artin-Tits groups, the classical monoids have no such Garside property. In the present paper we define dual monoids for all Artin-Tits groups and we prove that for the type \tilde{A}_n we get a (quasi)-Garside structure. Such a structure provides normal forms for the Artin-Tits group elements and allows to solve some questions such as to determine the centralizer of a power of the Coxeter element in the Artin-Tits group.

More precisely, if W is a Coxeter group, one can consider the length l_R on W with respect to the generating set R consisting of all reflections. Let c be a Coxeter element in W and let P_c be the set of elements $p \in W$ such that c can be written $c = pp'$ with $l_R(c) = l_R(p) + l_R(p')$. We define the monoid $M(P_c)$ to be the monoid generated by a set \underline{P}_c in one-to-one correspondence, $p \mapsto \underline{p}$, with P_c with only relations $\underline{pp'} = \underline{p}.\underline{p'}$ whenever p, p' and pp' are in P_c and $l_R(pp') = l_R(p) + l_R(p')$. We conjecture that the group of quotients of $M(P_c)$ is the Artin-Tits group associated to W and that it has a simple presentation (see 1.1 (ii)). These conjectures are known to be true for spherical type Artin-Tits groups. Here we prove them for Artin-Tits groups of type \tilde{A} . Moreover, we show that for exactly one choice of the Coxeter element (up to diagram automorphism) we obtain a (quasi-) Garside monoid. The proof makes use of non-crossing paths in an annulus which are the counterpart in this context of the non-crossing partitions used for type A .

MSC:

- 20F36 Braid groups; Artin groups
- 20F05 Generators, relations, and presentations of groups
- 20M05 Free semigroups, generators and relations, word problems
- 20F55 Reflection and Coxeter groups (group-theoretic aspects)
- 57M07 Topological methods in group theory

Cited in **20** Documents

Keywords:

Artin-Tits groups of spherical type; Garside structures; Artin monoids; dual monoids; normal form theorems; centralizers; Coxeter elements; Coxeter groups; generating sets of reflections; groups of quotients

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