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**Tree-depth, subgraph coloring and homomorphism bounds.** (English) Zbl 1089.05025  
Eur. J. Comb. 27, No. 6, 1022-1041 (2006).

Summary: We define the notions tree-depth and upper chromatic number of a graph and show their relevance to local-global problems for graph partitions. In particular we show that the upper chromatic number coincides with the maximal function which can be locally demanded in a bounded coloring of any proper minor closed class of graphs. The rich interplay of these notions is applied to a solution of bounds of proper minor closed classes satisfying local conditions. In particular, we prove the following result: For every graph  $M$  and finite set  $\mathcal{F}$  of connected graphs there exists a (universal) graph  $U = U(M, \mathcal{F}) \in \text{Forb}_h(\mathcal{F})$  such that any graph  $G \in \text{Forb}_h(\mathcal{F})$  which does not have  $M$  as a minor satisfies  $G \rightarrow U$  (i.e. is homomorphic to  $U$ ).

This solves the main open problem of restricted dualities for minor closed classes and as an application it yields the bounded chromatic number of exact odd powers of any graph in an arbitrary proper minor closed class. We also generalize the decomposition theorem of *M. DeVos, G. Ding, B. Oporowski, D. P. Sanders, B. Reed, P. Seymour, and D. Vertigan* [J. Comb. Theory, Ser. B 91; 25–41 (2004; [Zbl 1042.05036](#))].

**MSC:**

[05C15](#) Coloring of graphs and hypergraphs  
[05C83](#) Graph minors

Cited in **6** Reviews  
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chromatic number; graph partitions; minor closed class; decomposition

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