Caffarelli, L. A.; **Lee, K.-A.**; **Mellet, A.**

Homogenization and flame propagation in periodic excitable media: the asymptotic speed of propagation. (English) Zbl 1093.35010

Commun. Pure Appl. Math. 59, No. 4, 501-525 (2006).

The authors describe some qualitative properties of a flame propagation problem posed in a periodic medium. They indeed consider the problem $\partial_t u = \Delta u - f(x/\varepsilon)\beta_\delta(u)$ posed in $\mathbb{R}^n \times \mathbb{R}$. Here f is a 1-periodic function in all directions which satisfies $0 < \lambda \leq f(x) \leq \Lambda$. β_δ is deduced from a Lipschitz continuous function β through $\beta_\delta(s) = \beta(s/\delta)/\delta$, β being positive in $(0,1)$, equal to 0 elsewhere, and increasing on $[0,b]$ for some positive b. The small parameters ε and δ are linked through $\delta = \varepsilon\tau$, for some positive τ. The authors consider pulsating travelling fronts $u^{\varepsilon,\delta}$ for the above problem, that is solutions satisfying $u \to 0$ (resp. 1) as $x \cdot e \to -\infty$ (resp. $+\infty$), where $e \in S^{n-1}$, and $u(x+k,t) = u(x, t - k\cdot e/c^{\varepsilon,\delta}(e))$, for some real $c^{\varepsilon,\delta}(e)$. *H. Berestycki* and *F. Hamel* proved in [Commun. Pure Appl. Math. 55, No. 8, 949–1032 (2002; Zbl 1024.37054)] some existence and uniqueness result for $c^{\varepsilon,\delta}(e)$ and $u^{\varepsilon,\delta}$. Upper and lower bounds are given for $\gamma^{\varepsilon,\delta}(e) = c^{\varepsilon,\delta}(e)$ in terms of planelike solutions of the associated stationary problem. Given $\eta > 0$, the main result of the paper proves that the slope $\gamma^{\varepsilon,\delta}(e)$ belongs to $(\gamma^\tau_{\min}(e) - \eta, \gamma^\tau_{\min}(e) + \eta)$, when ε is small enough. When τ goes to 0, the quantity $\gamma^\tau_{\min}(e)$ converges to some $\gamma_{\min}(e)$ which can be computed in a few number of special cases, among which is the 1D case. The authors here extend previous results they obtained in [Arch. Ration. Mech. Anal. 172, No. 2, 153–190 (2004; Zbl 1058.76070)]. The proof is based on the qualitative properties of planelike solutions.

Reviewer: Alain Brillard (Mulhouse)

MSC:

35B27 Homogenization in context of PDEs; PDEs in media with periodic structure

35K55 Nonlinear parabolic equations

80A25 Combustion

Keywords:

Planelike solution; Upper and lower bounds

Full Text: DOI

References:

[1] ; ; Uniform estimates for regularization of free-boundary problems. Analysis and partial differential equations, 567–619. Lectures Notes in Pure and Applied Mathematics, 122. Dekker, New York, 1990.

[2] Berestycki, Comm Pure Appl Math 55 pp 949– (2002)

[3] Caffarelli, Differential Integral Equations 8 pp 1585– (1995)

[4] Caffarelli, Comm Pure Appl Math 54 pp 1403– (2001)

[5] Caffarelli, Amer J Math 120 pp 391– (1998)

[6] ; Homogenizations of nonvariational viscosity solutions. Preprint, 2005.

[7] ; Homogenization of the osciillating free boundaries: the elliptic case. Preprint, 2005.

[8] Caffarelli, Arch Ration Mech Anal 172 pp 153– (2004)

[9] Daskalopoulos, Duke Math J 108 pp 295– (2001)

[10] Daskalopoulos, Comm Pure Appl Math 55 pp 633– (2002)

[11] Daskalopoulos, Comm Partial Differential Equations 29 pp 71– (2004)