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**The growth of solutions of linear differential equations with coefficients of iterated order in the unit disc.** (English) Zbl 1105.34059

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The authors study the meromorphic solutions of a linear differential equation in the unit disc. They first give a definition of iterated order of a meromorphic function in the unit disc. They define the iterated $n$th-order $\sigma_n(f)$ of the meromorphic function $f$ as

$$\sigma_n(f) = \limsup_{r\to\infty} \frac{\log^{[n]} T(r,f)}{\log \frac{1}{1-r}}, \quad n \in \mathbb{N},$$

and the growth index of the iterated order of a meromorphic function $f(z)$ in the unit disc as

$$i(f) = \begin{cases} 0 & \text{if } f \text{ is nonadmissible;} \\ \min\{n : \sigma_n(f) < \infty\} & \text{if } f \text{ is admissible;} \\ \infty & \text{if } \sigma_n(f) = \infty \text{ for all } n \in \mathbb{N}. \end{cases}$$

Then, they investigate the iterated order of analytic solutions of the following linear differential equation with analytic coefficients in the unit disc

$$f^{(k)} + a_{k-1}(z) f^{(k-1)} + \cdots + a_0(z) f = 0.$$

For example, they prove that if $f$ is an analytic solution of this equation, then $i(f) \le \max\{i(a_j),\ j = 1, \cdots, k-1\}$.

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**MSC:**

34M10 Oscillation, growth of solutions to ordinary differential equations in the complex domain
30D30 Meromorphic functions of one complex variable, general theory



**Keywords:**

meromorphic function in the unit disc; linear differential equation

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