

Fayad, Bassam; Katok, Anatole

Constructions in elliptic dynamics. (English) Zbl 1089.37012
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Stable asymptotic behavior of conservative dynamical systems can be represented by quasiperiodic motions on tori: either translations by a vector α or linear flows with vector frequency ω . The motion is said to be of Diophantine type when α or ω is not well-approximated by rational numbers, and of Liouvillean type when good rational approximations exist. For dynamical systems of Diophantine type, orbital portraits are generally stable under small perturbations. However, systems of Liouvillean type exhibit complex instabilities when perturbed but usually only after long periods of time.

This paper develops some new applications of a constructive method called approximation by conjugation for creating Liouvillean-type examples of exotic C^∞ volume-preserving diffeomorphisms of a compact manifold. The technique was originally developed by *D. V. Anosov* and *A. B. Katok* [Trans. Mosc. Math. Soc. 23(1970), 1–35 (1972; [Zbl 0255.58007](#))]. Several variations of approximation by conjugation are described. The general scheme uses nontrivial circle actions on a differential manifold that preserve a smooth volume. Volume-preserving maps with interesting and surprising topological properties are obtained as the limit of volume-preserving periodic transformations. The authors apply the approximation by conjugation method to pose and investigate questions about the ergodic properties of volume-preserving diffeomorphisms. The technique they describe is a significant tool for constructing examples of smooth dynamical systems with prescribed ergodic or topological properties. Several open problems are posed.

Reviewer: [William J. Satzer jun. \(St. Paul\)](#)

MSC:

- [37C05](#) Dynamical systems involving smooth mappings and diffeomorphisms
- [37C40](#) Smooth ergodic theory, invariant measures for smooth dynamical systems
- [37E99](#) Low-dimensional dynamical systems

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Keywords:

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