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An equivariant foliated version of Brouwer's translation theorem. (Une version feuilletée équivariante du théorème de translation de Brouwer.) (French. English summary)

Zbl 1152.37015

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L. E. J. Brouwer's plane translation theorem [Math. Ann. 72, 37–54 (1912; JFM 43.0569.02)] tells us that for a fixed-point-free orientation-preserving homeomorphism f of the plane, every point belongs to a proper topological embedding C of $\mathbb{R}$ (the so-called Brouwer lines), disjoint from its image and separating $f(C)$ and $f^{-1}(C)$ (more recent proofs of Brouwer's theorem are available in, for instance, [*L. Guillou*, Topology 33, 331–351 (1994; Zbl 0924.55001)] or in [*J. Franks*, Ergodic Theory Dyn. Syst. 12, 217–226 (1992; Zbl 0767.58025)].

The main result of the paper under review is an equivariant foliated version of Brouwer's theorem: Let G be a discret group of orientation preserving homeomorphisms acting freely and properly on the plane. If f is a homeomorphism the Brouwer which commutes with the elements of G, then there exists a G-invariant topological foliation of the plane by Brouwer lines. The previous result is applied in several ways, for instance, in the framework of area-preserving surface homeomorphisms, the author obtains a new proof of Franks' theorem [*J. M. Franks*, New York J. Math. 2, 1–19, electronic (1996; Zbl 0891.58033)] which says that area-preserving two-sphere homeomorphisms having at least three fixed points always have an infinite number of periodic orbits. Another application is the following result: any Hamiltonian homeomorphism of a closed surface of genus greater or equal to 1 has infinitely many contractible periodic points.

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MSC:

37E30	Dynamical systems involving homeomorphisms and diffeomorphisms of planes and surfaces	Cited in **29** Documents
37C25	Fixed points and periodic points of dynamical systems; fixed-point index theory, local dynamics	
37C85	Dynamics induced by group actions other than $\mathbb{Z}$ and $\mathbb{R}$, and $\mathbb{C}$	
37J10	Symplectic mappings, fixed points (dynamical systems) (MSC2010)	
54H20	Topological dynamics (MSC2010)	

Keywords:

Brouwer's plan translation theorem; Brouwer line; Brouwer homeomorphism; foliation; area-preserving homeomorphism; orientation preserving homeomorphism

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References:

[1] G. D. Birkhoff, An extension of Poincaré's last geometric theorem, Acta Math., 47 (1925), 297–311.

[2] L. E. J. Brouwer, Beweis des ebenen Translationssatzes, Math. Ann., 72 (1912), 37–54. · Zbl 43.0569.02

[3] M. Brown, A new proof of Brouwer's lemma on translation arcs, Houston J. Math., 10 (1984), 35–41. · Zbl 0551.57005

[4] M. Brown, E. Slaminka, and W. Transue, An orientation preserving fixed point free homeomorphism of the plane which admits no closed invariant line, Topol. Appl., 29 (1988), 213–217. · Zbl 0668.54024

[5] P. H. Carter, An improvment of the Poincaré-Birkhoff theorem, Trans. Am. Math. Soc., 269 (1982), 285–299. · Zbl 0507.55002

[6] C. Conley, Isolated invariants sets and Morse index, CBMS Reg. Conf. Series in Math., Am. Math. Soc., 38 (1978). · Zbl 0397.34056

[7] A. Fathi, An orbit closing proof of Brouwer's lemma on translation arcs, Enseign. Math., 33 (1987), 315–322. · Zbl 0649.54022

[8] A. Floer, Proof of the Arnold conjectures for surfaces and generalizations to certain Kähler manifolds, Duke Math. J., 51 (1986), 1–32. · Zbl 0607.58016

[9] M. Flucher, Fixed points of measure preserving torus homeomorphism, Manuscr. Math., 68 (1990), 271–293. · Zbl 0722.58027

[10] J. Franks, Generalizations of the Poincaré-Birkhoff theorem, Ann. Math., 128 (1988), 139–151. · Zbl 0676.58037

[11] J. Franks, Recurrence and fixed points of surface homeomorphisms, Ergodic Theory Dyn. Syst., 8* (1988), 99–107. · Zbl 0634.58023

[12] J. Franks, A new proof of the Brouwer plane translation theorem, Ergodic Theory Dyn. Syst., 12 (1992), 217–226. · Zbl 0767.58025

[13] J. Franks, Geodesics on S2 and periodic points of annulus homeomorphisms, Invent. Math., 108 (1992), 403–418. · Zbl 0766.53037

[14] J. Franks, Area preserving homeomorphisms of open surfaces of genus zero, New York J. Math., 2 (1996), 1–19. · Zbl 0891.58033

[15] J. Franks, The Conley index and non-existence of minimal homeomorphisms, Ill. J. Maths, 43 (1999), 457–464. · Zbl 0948.37005

[16] J. Franks and M. Handel, Periodic points of Hamiltonian surface diffeomorphims, Geom. Topol., 7 (2003), 713–756. · Zbl 1034.37028

[17] J.-M. Gambaudo et P. Le Calvez, Infinité d'orbites périodiques pour les difféomorphismes conservatifs de l'anneau, Mém. Soc. Math Fr., 79 (1999), 125–146.

[18] L. Guillou, Théorème de translation plane de Brouwer et généralisations du théorème de Poincaré-Birkhoff, Topology, 33 (1994), 331–351.

[19] M.-E. Hamstrom, The space of homeomorphisms on a torus, Ill. J. Math., 9 (1965), 59–65. · Zbl 0127.13505

[20] M.-E. Hamstrom, Homotopy groups of the space of homeomorphisms on a 2-manifold, Ill. J. Math., 10 (1966), 563–573. · Zbl 0151.33002

[21] M. Handel, There are no minimal homeomorphisms of the multipunctured plane, Ergodic Theory Dyn. Syst.,12 (1992), 75–83. · Zbl 0769.58037

[22] M. Handel, A fixed point theorem for planar homeomorphisms, Topology, 38 (1999), 235–264. · Zbl 0928.55001

[23] M. Handel and W. P. Thurston, New proofs of some results of Nielsen, Adv. Math., 56 (1985), 173–191. · Zbl 0584.57007

[24] H. Hofer and E. Zehnder, Symplectic invariants and Hamiltonian dynamics, Birkhäuser, Basel–Boston–Berlin (1994). · Zbl 0805.58003

[25] B. de Kerékjártó, The plane translation theorem of Brouwer and the last geometric theorem of Poincaré, Acta Sci. Math. Szeged, 4 (1928–29), 86–102.

[26] B. Jiang, Lectures on Nielsen Fixed Point Theory, Contemp. Math., 14 (1983). · Zbl 0512.55003

[27] P. Le Calvez, Propriétés dynamiques de l'anneau et du tore, Astérisque, 204 (1991). · Zbl 0784.58033

[28] P. Le Calvez, Une généralisation du théorème de Conley-Zehnder aux homéomorphismes du tore de dimension deux, Ergodic Theory Dyn. Syst., 17 (1997), 71–86. · Zbl 0877.58011

[29] P. Le Calvez, Une propriété dynamique des homéomorphismes du plan au voisinage d'un point fixe d'indice > 1, Topology, 38 (1999), 25–35.

[30] P. Le Calvez, Dynamique des homéomorphismes du plan au voisinage d'un point fixe, Ann. Sci. Éc. Norm. Supér., IV Sér., 36 (2003), 139–171. · Zbl 1017.37017

[31] P. Le Calvez, Une version feuilletée du théorème de translation de Brouwer, Comment. Math. Helv., 21 (2004), 229–259. · Zbl 1073.37045

[32] P. Le Calvez et A. Sauzet, Une démonstration dynamique du théorème de translation de Brouwer, Expo. Math., 14 (1996), 277–287. · Zbl 0859.54029

[33] P. Le Calvez et J.-C. Yoccoz, Suite des indices de Lefschetz des itérés pour un domaine de Jordan qui est un bloc isolant. Manuscrit.

[34] F. Le Roux, Homéomorphismes de surfaces, théorème de la fleur de Leau-Fatou et de la variété stable, Astérisque, 292 (2004).

[35] S. Matsumoto, Arnold conjecture for surface homeomorphisms, Proceedings of the French–Japanese Conference "Hyperspace Topologies and Applications" (La Bussière, 1997), Topol. Appl., 104 (2000), 191–214. · Zbl 0974.37040

[36] N. A. Nikishin, Fixed points of diffeomorphisms of two-dimensional spheres preserving an oriented plane, Funkts. Anal. Prilozh., 8 (1974), 84–85. · Zbl 0298.57019

[37] S. Pelikan and E. Slaminka, A bound for the fixed point index of area-preserving homeomorphisms of two-manifolds, Ergodic Theory Dyn. Syst., 7 (1987), 463–479. · Zbl 0632.58014

[38] D. Salamon and E. Zehnder, Morse theory for periodic solutions of Hamiltonian systems and the Maslov index, Commun. Pure Appl. Math., 45 (1992), 1303–1360. · Zbl 0766.58023

[39] A. Sauzet, Application des décompositions libres à l'étude des homéomorphismes de surface, Thèse de l'Université Paris 13 (2001).

[40] M. Schwarz, On the action spectrum for closed symplectically aspherical manifolds, Pac. J. Math., 193 (2000), 419–461. · Zbl 1023.57020

[41] S. Schwartzman, Asymptotic cycles, Ann. Math., 68 (1957), 270–284. · Zbl 0207.22603

[42] J.-C. Sikorav, Points fixes d'une application symplectique homologue à l'identité, J. Differ. Geom., 22 (1985), 49–79. · Zbl 0555.58013

[43] C. P. Simon, A bound for the fixed-point index of an area-preserving map with applications to mechanics, Invent. Math., 26

(1974), 187–200. · Zbl 0331.55006