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A posteriori error estimates of discontinuous Galerkin methods for non-standard Volterra integro-differential equations. (English) Zbl 1094.65135

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The authors begin by citing earlier literature relating to the use of the discontinuous Galerkin (DG) method. They analyze the DG method for three classes of Volterra integro-differential equations (VIDEs); linear, semilinear with Hammerstein-type memory term and non-linear with non-standard memory term. An introduction to the DG method is included.

A theorem giving a posteriori error estimates for these VIDEs is stated and proved. The proof of the theorem considers each type of VIDE separately. For the linear type the dual problem is introduced and its stability is established by stating and proving a lemma. A theorem describing the effectivity of the a posteriori error estimates is stated and an indication of the method of proof is given.

The authors then analyze the error of the 'discretized' DG(m) approximation to the VIDE for the linear case and state that a similar analysis can be carried out for semilinear and non-standard VIDEs. A theorem is stated and proved. The proof makes use of the Peano theorem for quadrature and a discrete Gronwall lemma. The authors comment that this theorem applies to the class of VIDE with completely monotonic kernels.

The authors conclude the paper with illustrative numerical examples, one for each type of VIDE. For the linear type, the DG errors and the discretized DG errors are compared graphically to the exact solution and the adaptive mesh based on the a posteriori error estimators is plotted. For the semilinear, scalar, VIDE the DG errors are compared on uniform and adaptive meshes. A table of results is included to support the comment that the DG method is more effective on an adaptive mesh than on a uniform mesh. In the example involving a non-standard type results are presented which, using the ratio of the a posteriori error and the a priori error to measure the effectivity of the a posteriori error estimate, indicate that the a posteriori error estimators are reliable when used for the selection of an adaptive mesh.

Reviewer: Pat Lumb (Chester)

MSC:

65R20	Numerical methods for integral equations
45G10	Other nonlinear integral equations
45J05	Integro-ordinary differential equations

Keywords:

discontinuous Galerkin method; linear, semilinear and non-standard Volterra integro-differential equations; a posteriori error estimates; stability; numerical examples

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