Koganov, L. M.

Application of the notion of pseudogenerated two-index sequence and lemmas on substitution. (Russian) Zbl 0733.05004

Komb. Anal. 8, 139-153 (1989).

The two-index sequence { P(n,k)} is said to be pseudogenerable of order p if there are (in a neighbourhood of zero) two analytic functions ϕ (z) and ψ (z) such that ψ (0)$\neq$ 0, $\phi(0) = \phi'(0) = ... = \phi^{(p-1)}(0) = 0$, $\phi^{(p)}(0) \neq 0$, $\sum_{n=0}^{\infty} P(n,k)z^n/n! = \psi(z)\phi(z))^k/k!$ for $k \geq 0$. Substitution lemma: Let { P(n,k)} be pseudogenerable with associated ϕ (z) and ψ (z) and let $f(t) = \sum_{n=0}^{\infty} a_n(t^n/n!)$ be the exponential generating function of the sequence $\{a_n\}$. Then ψ (z)$\cdot f(\phi(z))$ is the exponential generating function of the sequence $\{P_n\}$, where $P_n = \sum_{k=0}^{n} P(n,k) \cdot a_k$, $n \geq 0$. The author develops a theory based on this lemma which yields in a nice way many well-known identities in combinatorics. In particular, he gives formulas for the convolution of pseudogenerable sequences and for the inverse (relative to convolution) of a pseudogenerable sequence of order 1.

MSC:

05A15 Exact enumeration problems, generating functions
05A19 Combinatorial identities, bijective combinatorics

Keywords:

exponential generating function