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Logical equations and admissible rules of inference with parameters in modal provability logics. (English) Zbl 0729.03012

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The aim of this paper is to study admissible inference rules for the modal provability logics GL and S. It is proved that none of these logics has a basis for admissible rules in a finite number of variables, in particular, they do not have finite bases. It is proved that GL and S are decidable by admissibility, some algorithms are found which recognize admissibility of usual inference rules and inference rules in generalized form - inference rules with parameters (or metavariables). By using recognizability of admissibility of inference rules with parameters, we can recognize solvability of logical equations in GL and S and construct some of their solutions. Thus, the analogues of H. Friedman's problem for GL and S are affirmatively solved, the analogues of A. Kuznetsov's problem of finiteness of a basis for admissible rules for GL and S have negative solutions, and the problems of solvability of logical equations in GL and S have positive solutions.

Reviewer: [V.V.Rybakov](#)

MSC:

- [03B45](#) Modal logic (including the logic of norms)
- [03F40](#) Gödel numberings and issues of incompleteness
- [03B25](#) Decidability of theories and sets of sentences

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Keywords:

Gödel-Löb logic; admissible inference rules; modal provability logics; algorithms; solvability of logical equations

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