Bestvina, M.; **Feighn, M.**

A combination theorem for negatively curved groups. (English) Zbl 0724.57029

J. Differ. Geom. 35, No. 1, 85-102 (1992); addendum and correction ibid. 43, No. 4, 783-788 (1996).

In this important paper the following issue is addressed: When is a result of gluing negatively curved spaces negatively curved? Previously the gluing constructions were carried out in the category of negatively (or non-positively) curved spaces in the sense of A. D. Aleksandrov, the gluing maps were isometries and most of the time gluing was carried along a totally geodesic subspace. In the present paper the spaces are hyperbolic in the coarse sense of M. Gromov, gluings are of a rather flexible kind (on the level of universal covering they induce quasi-isometries) and an additional natural hypothesis (annuli flare condition) is satisfied. These conditions guarantee that the resulting space is negatively curved.

This setting has been dictated by applications, and indeed the paper contains wonderful nontechnical corollaries. Let us list just two: (1) The mapping torus of an automorphism f of a negatively curved group is negatively curved if and only if f is hyperbolic, that is, if there are m and $\lambda > 0$, such that for all g in G, $\lambda|g| \leq \max\{|f^m g|, |f^{-m} g|\}$. This includes as special cases automorphisms of free groups, where hyperbolicity translates into having no nontrivial periodic conjugacy classes, and automorphisms of surface groups where it translates to pseudo-Anosov. (2) The free products and HNN extensions of negatively curved groups over virtually cyclic subgroups are negatively curved if and only if the resulting group contains no Baumslag-Solitar groups.

The technical result is strong and natural and the applications beautiful, but there are still issues about gluing of spaces that should eventually be addressed. First of all the paper deals only with graphs of spaces. More naive isometric gluings work for more complicated gluing patterns (complexes of spaces) and ideally one should try to study these in the setup proposed by the authors. Second, we know some intriguing examples of bundles over S^1, obtained from ramified coverings of tori, described in a paper by *M. Gromov* [Essays in group theory, Publ., Math. Sci. Res. Inst. 8, 75–263 (1987; Zbl 0634.20015)]. One may wonder what properties of the gluing map – the holonomy – make the total space negatively curved even though the fibers are not negatively curved.

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57M50 General geometric structures on low-dimensional manifolds

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