Ramshaw, Lyle
Blossoms are polar forms. (English) Zbl 0705.65008
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Summary: Consider the functions $H(t) := t^2$ and $h(u, v) := uv$. The identity $H(t) = h(t, t)$ shows that H is the restriction of h to the diagonal $u = v$ in the uv-plane. Yet, in many ways, a bilinear function like h is simpler than a homogeneous quadratic function like H. More generally, if F(t) is some n-ic polynomial function, it is often helpful to study the polar form of F, which is the unique symmetric, multiaffine function $f(u_1, ..., u_n)$ satisfying the identity $F(t) = f(t, ..., t)$. The mathematical theory underlying splines is one area where polar forms can be particularly helpful, because two pieces F and G of an n-ic spline meet at a point r with C^k parametric continuity if and only if their polar forms f and g agree on all sequences of n arguments that contain at least n-k copies of r.

This polar approach to the theory of splines emerged in rather different guises in three independent research efforts: Paul de Faget de Casteljau called it 'shapes through poles'; Carl de Boor called it 'B-splines without divided differences'; and the author called it 'blossoming'. This paper reviews the work of de Casteljau, de Boor, and the author in an attempt to clarify the basic principles that underly the polar approach. It also proposes a consistent system of nomenclature as a possible standard.

MSC:

65D07 Numerical computation using splines
41A15 Spline approximation

Keywords:

affine interpolation; B-spline; Bézier point; blossoming; de Boor algorithm; de Casteljau algorithm; dual functional; homogeneity; multiaffine function; polar form; quasi-interpolant; spline reproductivity; tensor

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References:

[1] Bartels, R.H.; Beatty, J.C.; Barsky, B.A., An introduction to splines for use in computer graphics and geometric modeling, (1987), Morgan Kaufmann 95 First Street, Los Altos, CA · Zbl 0682.65003
[2] Boˆcher, M., (), 114-128
[3] Boehm, W., Smooth curves and surfaces, (), 175-184
[4] Boehm, W.; Farin, G.; Kahmann, J., A survey of curve and surface methods in CAGD, Computer aided geometric design, 1, 1-60, (1984) · Zbl 0604.65005
[5] Catmull, E.; Rom, R., A class of local interpolating splines, (), 317-326
[6] Chui, C.K., Multivariate splines, (1988), SIAM Philadelphia, PA · Zbl 0644.41007
[7] de Boor, C., A practical guide to splines, (1978), Springer Berlin · Zbl 0406.41003
[8] de Boor, C., B(asic)-spline basics, first portion of course notes for course #5 at ACM SIGGRAPH '86, (1986)
[9] de Boor, C.; Höllig, K., B-splines without divided differences, (), 21-27
[10] de Casteljau, P., ()
[11] Farin, G., Curves and surfaces for computer aided geometric design: A practical guide, (1988), Academic Press New York · Zbl 0694.68004
[12] Graham, R.L.; Knuth, D.E.; Patashnik, O., (), 47-48
[13] Lang, S., Algebra, (1984), Addison-Wesley Reading, MA
[14] Lee, E.T.Y., Some remarks concerning B-splines, Computer aided geometric design, 2, 307-311, (1985) · Zbl 0606.65004
[15] Ramshaw, L., Blossoming: a connect-the-dots approach to splines, Digital equipment corp. systems research center technical report #19, (1987)
[16] Ramshaw, L., Béziers and B-splines as multiaffine maps, (), 757-776
[17] Ramshaw, L., Blossoms are polar forms, Digital equipment corp. systems research center technical report #34, (1989) · Zbl

0705.65008

[18] Ramshaw, L., Suitening our nomenclature, SIGACT news, 70, 60-61, (1989)

[19] Salmon, G., (), 49-52

[20] Schumaker, L.L., (), 262

[21] Seidel, H.P., A new multiaffine approach to B-splines, Computer aided geometric design, 6, 23-32, (1989) · Zbl 0666.65011

[22] Seidenberg, A.; Seidenberg, A., (), 184-193

[23] van der Waerden, B.L., (), 20, translated from the German fifth edition

