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**Analysis of resonances in the spin-orbit problem in celestial mechanics. I: The synchronous resonance.** (English) Zbl 0699.70014

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Summary: We study the stability of spin-orbit resonances in celestial mechanics, namely the exact commensurabilities between the periods of rotation and revolution of satellites or planets. We introduce a mathematical model describing an approximation of the physical situation and we select a set of satellites for which such simplified model provides a good approximation. Applying the Kolmogorov-Arnold-Moser theory we are able to construct invariant surfaces trapping the synchronous resonance from above and below. The existence of such surfaces, established for the natural values of the physical and orbital parameters, allows to prove the stability of the 1:1 resonance. Furthermore we try to construct KAM tori with frequencies as close as possible to one so to trap the synchronous resonance in a finer region.

**MSC:**

70F15 Celestial mechanics
70K20 Stability for nonlinear problems in mechanics
70M20 Orbital mechanics
70-08 Computational methods for problems pertaining to mechanics of particles and systems



**Keywords:**

stability of spin-orbit resonances; celestial mechanics; periods of rotation; revolution of satellites; planets; Kolmogorov-Arnold-Moser theory; invariant surfaces; synchronous resonance; KAM tori

**Full Text:** DOI

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