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A proof of the theorem characterizing the generalized J-homomorphism. (English)

Zbl 0695.55010
Homotopy theory and related topics, Proc. Int. Conf., Kinosaki/Japan 1988, Lect. Notes Math. 1418, 95-104 (1990).

[For the entire collection see Zbl 0685.00018.]

Let S^n be the unit sphere, $\Omega^k S^n$ for$k \neq n$ the k-fold loop space of S^n which is identified with a space of based maps from S^k to S^n. Let $V_{n,k}$ be the Stiefel manifold. One identifies $V_{n,k}$ with a space of normed linear maps from $\mathbb{R}^k$ to $\mathbb{R}^n$. Then $V_{k,n}$, acting on S^k as $\mathbb{R}^k$ with a point at infinity, is considered as a subspace of $\Omega^k S^n$. This defines an inclusion $j_{n,k} : V_{n,k} \to \Omega^k S^n$. The induced map in homotopy is called the generalized J-map and is denoted by

$$J_{n,k} : [X, V_{n,k}] \to \left[X, \Omega^k S^n\right] \approx \left[\Sigma^k X, S^n\right].$$

One denotes by $\partial : [\Sigma X, V_{n,k}] \to \left[X, S^{n-k-1}\right]$ for $k+1 \leq n$ the connecting map induced from the canonical fibration $p : V_{n,k+1} \to V_{n,k}$. Then the main result of the paper is:

Theorem. There is a commutative diagram up to sign:

$$\begin{array}{ccccc} [\Sigma X, V_{n,k}] & @> & J_{n,k} & >> & \left[\Sigma^{k+1} X, S^n\right] \\ & {\scriptstyle\partial} \searrow & & \nearrow {\scriptstyle \Sigma^{k+1}} & \\ & & \left[X, S^{n-k-1}\right] & & \end{array}$$

In the case of $X = S^r$, various proofs are known [*I.M. James*, The topology of Stiefel manifolds (1976; Zbl 0337.55017)]. The purpose of the present note is to give a homotopy-theoretic proof using a method different from that of *B. Gray* [J. Lond. Math. Soc., II. Ser. 16, 124-130 (1977; Zbl 0396.55014)].

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MSC:

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55P35 Loop spaces

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generalized J-homomorphism; sphere; loop space; Stiefel manifold