

Conrad, Paul F.; Martinez, Jorge

Locally finite conditions on lattice-ordered groups. (English) Zbl 0688.06011
Czech. Math. J. 39(114), No. 3, 432-444 (1989).

Suppose α stands for a certain property or class of ℓ -groups. We say that an ℓ -group G is locally α if every finitely ℓ -generated ℓ -subgroup of G satisfies α . \mathcal{S} denotes the class of ℓ -groups which are ℓ -isomorphic to an ℓ -group of real-valued step functions, i.e. of functions $f \in \mathbb{R}^I$ with $f(I)$ finite. The authors study the following seven conditions: (1) Locally G satisfies the ACC on all subgroups. (2) Locally G satisfies the ACC on all ℓ -subgroups. (3) G is locally finitely generated. (4) Locally G has a finite root system of p prime subgroups. (5) Locally G has a finite basis. (6) G is locally finite-valued. (7) For each $0 < x \in G$, $G(x)/N_x \in \mathcal{S}$. It is shown that for a nilpotent ℓ -group G the conditions (1) through (7) are equivalent. Moreover some examples are given which indicate how some of the above implications can fail in general (e.g. (4) non \Rightarrow (1), (5) non \Rightarrow (4), (6) non \Rightarrow (5), (7) non \Rightarrow (6), (6) non \Rightarrow (3)). Also, they call attention to the ℓ -groups which are characterized by condition (7). Other problems are discussed. Finally, a list of questions is presented.

Reviewer: F.Šik

MSC:

06F15 Ordered groups
06F20 Ordered abelian groups, Riesz groups, ordered linear spaces

Cited in 2 Documents

Keywords:

local property; root system; finitely generated; lex-sum; free abelian group; step functions; nilpotent ℓ -group

Full Text: [EuDML](#)

References:

- [1] R. Baer: Representations of groups as quotient groups. Trans AMS, Vol. 58, (1946), 295-419. · [Zbl 0061.02703](#) · [doi:10.1090/S0002-9947-1945-0015107-1](#)
- [2] A. Bigard K. Keimel, S. Wolfenstein: Groupes et Anneaux Reticules. LNM 608, Springer; Berlin-Heidelberg-New York (1977). · [Zbl 0384.06022](#)
- [3] R. Bleier: Minimal vector lattice covers. Bul. Austral. Math. Soc., Vol. 5, (1971), 331-335. · [Zbl 0216.30502](#) · [doi:10.1017/S0004972700047286](#)
- [4] P. Conrad: Lattice-ordered Groups. Tulane University Lecture Notes (1970). · [Zbl 0258.06011](#)
- [5] P. Conrad: Minimal vector lattice covers. Bul. Austral. Math. Soc., Vol. 4, (1971), 35-39. · [Zbl 0199.34703](#) · [doi:10.1017/S0004972700046232](#)
- [6] P. Conrad: Epi-archimedean groups. Czech. Math. Journal, 24 (99), (1974), 192 - 218. · [Zbl 0319.06009](#) · [eudml:12783](#)
- [7] H. Hollister: Nilpotent (\setminus) -groups are representable. Alg. Univ. 8, (1978), 65-71. · [Zbl 0385.06024](#) · [doi:10.1007/BF02485371](#)
- [8] V. M. Kopytov: On lattice-ordered, locally nilpotent groups. Alg. & Log., (1975), 14, No. 4, 407-413.
- [9] J. Martinez: Torsion theory for lattice-ordered groups. Czech. Math. Jour., 25 (100), (1975), 284-299. · [Zbl 0321.06020](#) · [eudml:12865](#)
- [10] J. Martinez: Pairwise-splitting lattice-ordered groups. Czech. Math. Jour., 27 (102) (1977), 545-551. · [Zbl 0378.06009](#) · [eudml:13024](#)
- [11] J. Martinez: The fundamental theorem on torsion classes of lattice-ordered groups. Trans. AMS, Vol. 259, No. 1, May 1980, 311-317. · [Zbl 0433.06016](#) · [doi:10.2307/1998160](#)
- [12] D. J. S. Robinson: Finiteness conditions and generalized soluble groups. Part 1. Erg. d. Math., Band 62; Springer, Berlin-Heidelberg-New York, 1972. · [Zbl 0243.20033](#)
- [13] E. Weinberg: Free lattice-ordered abelian groups. Math. Ann. 151, (1963), 187-199. · [Zbl 0114.25801](#) · [doi:10.1007/BF01398232](#) · [eudml:161076](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.