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On the optimality and stability of competitive paths in continuous time growth models.
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J. Econ. Theory 48, No. 2, 526-547 (1989).

The purpose of the present paper is the extension of the results about the global asymptotic stability of stationary states to optimal control problems with a nonconcave Hamiltonian function.

After the introduction, in Section 2 the optimal control problem under consideration is defined and the new formulation of the sufficient conditions for a path to be optimal is given. This new formulation is based on the canonical transformations of modified Hamiltonian dynamical systems (MHDS) briefly described in Section 3. It is explained how to construct such transformations by means of generating functions and the sufficiency theorem from Section 2 is interpreted in terms of a canonical transformation.

Using a special canonical transformation with a quadratic generating function the main result of the paper in Section 4, is presented. It contains the generalization of three well-known conditions for the global asymptotic stability of stationary states of MHDS. The crucial part of the new conditions is the existence of a symmetric matrix such that a certain function defined in terms of the Hamiltonian and this matrix is concave.

Finally, Section 5 concludes with some remarks and possible generalizations of the presented results.

This well written paper provides an important contribution to the theory of optimal economic growth because of the applicability of the new stability conditions also in the presence of nonconvex technologies and production functions with increasing returns to scale. From the mathematical point of view, the method of canonical transformations might be useful also in obtaining other qualitative properties of MHDS.

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49K15	Optimality conditions for problems involving ordinary differential equations
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