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Recursive utility and optimal capital accumulation. I: Existence. (English) Zbl 0679.90014

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This paper demonstrates existence of optimal capital accumulation paths when the planner's preferences are represented by a recursive objective functional. Time preference is flexible. Mathematically, the problem is as follows:

$$\max imize \quad \int_0^\infty L(t,k,\dot{k}) \exp(\int_0^\infty R(s,k,\dot{k})ds)dt,$$

s.t. k: $\mathbb{R}_+ \to \mathbb{R}^m$ is an absolutely continuous function; $\dot{k} \in G(t,k)$ a.e.; $0 \leq k(0) \leq x$, where L: $\Omega \to \mathbb{R}_+$ and R: $\Omega \to \mathbb{R}$ are continuous on Ω and convex in k, which are the felicity function and the discounting function of the economy respectively; $\Omega \subset \mathbb{R} \times \mathbb{R}^m \times \mathbb{R}^m$ is the technology set of the economy and $G(t,k) = \{y : (t,k,y) \in \Omega\}$ is the investment correspondence, which is compact-convex- valued and upper semicontinuous in t. Existence of optimal paths is addressed via the classical Weierstrass theorem. An improved version of a lemma due to Varaiya proves compactness of the feasible set for the compact-open topology. A monotonicity argument is combined with a powerful theorem of Cesari to demonstrate upper semicontinuity.

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MSC:

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