Gurchenkov, S. A.
On the theory of varieties of lattice ordered groups. (English. Russian original) [Zbl 0679.20022]
Algebra Logic 27, No. 3, 153-167 (1988); translation from Algebra Logika 27, No. 3, 249-273 (1988).

Let the natural number $n = p_1^{n_1} p_2^{n_2} ... p_r^{n_r}$ be the product of prime numbers, where $p_1, p_2, ..., p_r$ are distinct prime numbers, $n_i \geq 1$ $(i = 1, 2, ..., r)$, $\bar{n} = n_1 + n_2 + ... + n_r + 1$ and $\mathfrak{L}_n$ be the ℓ- variety defined by the law $[x^n, y^n] = e$. In this paper the following main results are proved. 1) $\mathfrak{L}_n \subseteq (\mathfrak{A}_\ell)^{\bar{n}}$, where $\mathfrak{A}_\ell$ is the ℓ- variety of all abelian ℓ-groups (Theorem 1). 2) Let $\mathfrak{N}$ be an ℓ-variety and every linearly ordered group from the ℓ- variety $\mathfrak{N}$ is abelian. Then there exists a natural number $n = n(\mathfrak{N})$ such that $\mathfrak{N} \subseteq \mathfrak{L}_n$ (Theorem 2). 3) The lattice of all ℓ-subvarieties of the ℓ-variety $\mathfrak{L}_n \wedge (\mathfrak{A}_\ell)^2$ is described and it is proved: a) every ℓ-variety $\mathfrak{L} \subseteq \mathfrak{L}_n \wedge (\mathfrak{A}_\ell)^2$ has a finite basis of identities; b) if the ℓ-variety $\mathfrak{L}$ has finite basis rank, then the lattice of all ℓ-subvarieties of $\mathfrak{L}$ is finite (Theorems 6.7). 4) An ℓ-variety of nilpotent ℓ-groups of nilpotency class 3 with finite axiomatic rank and without independent basis of identities is constructed (Theorem 8). 5) The existence of linearly ordered nilpotent groups with the property $var_\ell G \neq var_\ell G^*$, where G^* is the Malcev completion of the nilpotent group G, is established (Theorem 9).

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MSC:

20E10	Quasivarieties and varieties of groups
20F60	Ordered groups (group-theoretic aspects)
06F15	Ordered groups
06B20	Varieties of lattices
20F18	Nilpotent groups
08B15	Lattices of varieties

Cited in **1** Document

Keywords:

lattice of ℓ-subvarieties; abelian ℓ-groups; ℓ-variety; finite basis of identities; nilpotent ℓ-groups; independent basis of identities; linearly ordered nilpotent groups; Malcev completion

Full Text: DOI

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