

**Gundersen, Gary G.**

**Finite order solutions of second order linear differential equations.** (English) Zbl 0669.34010  
Trans. Am. Math. Soc. 305, No. 1, 415-429 (1988).

We consider the differential equation  $f'' + A(z)f' + B(z)f = 0$  where  $A(z)$  and  $B(z)$  are entire functions. We find conditions on  $A(z)$  and  $B(z)$  which guarantee that every solution  $f \neq 0$  of the equation has infinite order. We also find conditions on  $A(z)$  and  $B(z)$  which guarantee that any finite order solution  $f \neq 0$  of the equation has not zero as a Borel exceptional value. We also show that if  $A(z)$  and  $B(z)$  satisfy certain growth conditions, then any finite order solution of the equation satisfies certain other growth conditions. Related results are also proven. Several examples are given to complement the theory.

Reviewer: [G.G.Gundersen](#)

**MSC:**

[34M99](#) Ordinary differential equations in the complex domain

Cited in **7** Reviews  
Cited in **46** Documents

**Keywords:**

[growth conditions](#); [examples](#)

**Full Text:** [DOI](#)

**References:**

- [1] Ichiro Amemiya and Mitsuru Ozawa, Nonexistence of finite order solutions of  $f^{(n)} + A(z)f' + B(z)f = 0$ , Hokkaido Math. J. 10 (1981), no. Special Issue, 1 – 17. · [Zbl 0554.34003](#)
- [2] Steven B. Bank, On the value distribution theory for entire solutions of second-order linear differential equations, Proc. London Math. Soc. (3) 50 (1985), no. 3, 505 – 534. · [Zbl 0545.30022](#) · [doi:10.1112/plms/s3-50.3.505](#) · [doi.org](#)
- [3] Steven B. Bank, Three results in the value-distribution theory of solutions of linear differential equations, Kodai Math. J. 9 (1986), no. 2, 225 – 240. · [Zbl 0635.34009](#) · [doi:10.2996/kmj/1138037205](#) · [doi.org](#)
- [4] Steven B. Bank and Ilpo Laine, On the oscillation theory of  $f^{(n)} + A(z)f' + B(z)f = 0$  where  $f$  is entire, Trans. Amer. Math. Soc. 273 (1982), no. 1, 351 – 363. · [Zbl 0505.34026](#)
- [5] Steven B. Bank and Ilpo Laine, Representations of solutions of periodic second order linear differential equations, J. Reine Angew. Math. 344 (1983), 1 – 21. · [Zbl 0524.34007](#)
- [6] Steven B. Bank and Ilpo Laine, On the zeros of meromorphic solutions and second-order linear differential equations, Comment. Math. Helv. 58 (1983), no. 4, 656 – 677. · [Zbl 0532.34008](#) · [doi:10.1007/BF02564659](#) · [doi.org](#)
- [7] Steven B. Bank, Ilpo Laine, and J. K. Langley, On the frequency of zeros of solutions of second order linear differential equations, Results Math. 10 (1986), no. 1-2, 8 – 24. · [Zbl 0635.34007](#) · [doi:10.1007/BF03322360](#) · [doi.org](#)
- [8] Steven B. Bank and J. K. Langley, On the oscillation of solutions of certain linear differential equations in the complex domain, Proc. Edinburgh Math. Soc. (2) 30 (1987), no. 3, 455 – 469. · [Zbl 0596.30049](#) · [doi:10.1017/S0013091500026857](#) · [doi.org](#)
- [9] A. S. Besicovitch, On integral functions of order  $\leq 1$ , Math. Ann. 97 (1927), no. 1, 677 – 695. · [Zbl 53.0294.05](#) · [doi:10.1007/BF01447889](#) · [doi.org](#)
- [10] M. L. Cartwright, Integral functions, Cambridge tracts in mathematics and mathematical physics, No. 44, Cambridge, at the University Press, 1956. · [Zbl 0075.05901](#)
- [11] M. Frei, Sur l'ordre des solutions entières d'une équation différentielle linéaire, C. R. Acad. Sci. Paris 236 (1953), 38 – 40 (French). · [Zbl 0050.08902](#)
- [12] Gary G. Gundersen, On the question of whether  $f^{(n)} + A(z)f' + B(z)f = 0$  can admit a solution  $f \equiv 0$  of finite order, Proc. Roy. Soc. Edinburgh Sect. A 102 (1986), no. 1-2, 9 – 17. · [Zbl 0598.34002](#) · [doi:10.1017/S0308210500014451](#) · [doi.org](#)
- [13] -, Estimates for the logarithmic derivative of a meromorphic function, plus similar estimates, J. London Math. Soc. (to appear). · [Zbl 0638.30030](#)
- [14] W. K. Hayman, Meromorphic functions, Oxford Mathematical Monographs, Clarendon Press, Oxford, 1964. · [Zbl 0115.06203](#)
- [15] Einar Hille, Ordinary differential equations in the complex domain, Wiley-Interscience [John Wiley & Sons], New York-London-Sydney, 1976. Pure and Applied Mathematics. · [Zbl 0343.34007](#)

- [16] J. K. Langley, On complex oscillation and a problem of Ozawa, *Kodai Math. J.* 9 (1986), no. 3, 430 – 439. · [Zbl 0609.34041](#) · [doi:10.2996/kmj/1138037272](#) · [doi.org](#)
- [17] A. I. Markushevich, *Theory of functions of a complex variable. Vol. II, Revised English edition translated and edited by Richard A. Silverman*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1965. · [Zbl 0142.32602](#)
- [18] Mitsuru Ozawa, On a solution of  $y^{(n)} + \dots + \lambda y = 0$ , *Kodai Math. J.* 3 (1980), no. 2, 295 – 309. · [Zbl 0463.34028](#)
- [19] Klaus Pöschl, Zur Frage des Maximalbetrages der Lösungen linearer Differentialgleichungen zweiter Ordnung mit Polynomkoeffizienten, *Math. Ann.* 125 (1953), 344 – 349 (German). · [Zbl 0050.31203](#) · [doi:10.1007/BF01343129](#) · [doi.org](#)
- [20] John Rossi, Second order differential equations with transcendental coefficients, *Proc. Amer. Math. Soc.* 97 (1986), no. 1, 61 – 66. · [Zbl 0596.30047](#)
- [21] Li-Chien Shen, Solution to a problem of S. Bank regarding exponent of convergence of zeros of the solutions of differential equation  $y^{(n)} + \dots = 0$ , *Kexue Tongbao* 30 (1985), no. 12, 1579 – 1585. · [Zbl 0636.34003](#)
- [22] G. Valiron, *Lectures on the general theory of integral functions*, translated by E. F. Collingwood, Chelsea, New York, 1949.
- [23] Hans Wittich, *Neuere Untersuchungen über eindeutige analytische Funktionen*, *Ergebnisse der Mathematik und ihrer Grenzgebiete (N.F.)*, Heft 8, Springer-Verlag, Berlin-Göttingen-Heidelberg, 1955 (German). · [Zbl 0067.05501](#)
- [24] H. Wittich, Subnormale Lösungen der Differentialgleichung:  $y^{(n)} + \dots + \lambda y = 0$ , *Nagoya Math. J.* 30 (1967), 29 – 37 (German). · [Zbl 0219.34005](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.