Wehler, Joachim

Hypersurfaces of the flag variety: Deformation theory and the theorems of Kodaira-Spencer, Torelli, Lefschetz, M. Noether and Serre. (English) Zbl 0662.14029

Math. Z. 198, No. 1, 21-38 (1988).

Let X denote a smooth hypersurface in the full flag manifold $\mathbb{F} \subseteq \mathbb{P}^n_{\mathbb{C}}$. The author proves that any small deformation of X is again a hypersurface of $\mathbb{F}$ provided the degree of X is at least 2 (3 if $n = 2$).

He also obtains a generalization of the classical Lefschetz theorem saying that $\pi_i(\mathbb{F}, X) = H_i(\mathbb{F}, X) = 0$ for $i \leq \dim(X) - p$, where p is computable from the degree of the embedding $X \in \mathbb{F}$. When $n = 3$, the author shows that the Picard number of X is 2, and as a consequence, every curve on X is in this case the variety of a global section of a 2-bundle on $\mathbb{F}$.

Reviewer: H.H.Andersen

MSC:

14M15 Grassmannians, Schubert varieties, flag manifolds
14D15 Formal methods and deformations in algebraic geometry
14J10 Families, moduli, classification: algebraic theory

Keywords:

smooth hypersurface in the full flag manifold; small deformation; Lefschetz theorem; Picard number

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