Keady, G.; **Kloeden, P. E.**

An elliptic boundary-value problem with a discontinuous nonlinearity. II. (English)

Zbl 0647.35029
Proc. R. Soc. Edinb., Sect. A 105, 23-36 (1987).

Summary: [For Part I, see the first author, ibid. 91, 161-174 (1981; Zbl 0511.35032).]

Let Ω be a bounded domain in $\mathbb{R}^2$. The study, begun in part I, of the boundary-value problem, for (λ /k,ψ),

$$-\Delta\psi \in \lambda H(\psi - k) \quad in \quad \Omega \subset \mathbb{R}^2, \quad \psi = 0 \quad on \quad \partial\Omega,$$

is continued. Here Δ denotes the Laplacian, H is the Heaviside step function and one of λ or k is a given positive constant. The solutions considered always have $\psi > 0$ in Ω and $\lambda/k > 0$, and have cores $A = \{(x, y) \in \Omega | \psi(x, y) > k\}$.

In the special case $\Omega = B(0, R)$, a disc, the explicit exact solutions of the branch τ_e have connected cores A and the diameter of A tends to zero when the area of A tends to zero. This result is established here for other convex domains Ω and solutions with connected cores A.

An adaptation of the maximum principles and of the domain folding arguments of *B. Gidas*, *W. M. Ni* and *L. Nirenberg* [Commun. Math. Phys. 68, 209-243 (1979; Zbl 0425.35020)] is an important step in establishing the above result.

MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
35R05 PDEs with low regular coefficients and/or low regular data
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

discontinuous nonlinearity; steady flow; inviscid incompressible fluid; compact vortex cores; Laplacian; Heaviside step function; maximum principles; domain folding arguments

Full Text: DOI

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