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Topology and duality in modal logic. (English) Zbl 0643.03014
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There are two major approaches to the semantics of propositional modal logics: the algebraic, as exemplified by modal algebras (i.e., Boolean algebras with an extra unary operation); and the relational, as exemplified by Kripke frames (i.e., sets with a binary operation). Each approach gives rise to an associated first order language, and much research in this area has centered around completeness theorems roughly of the form: sentence σ is a theorem of a set Γ of sentences relative to such-and-such a modal logic if and only if σ is a consequence of Γ in every modal algebra (or frame) that satisfies such-and-such (perhaps first order) conditions. Of course there are many variations on this theme, and it is natural to assume any general theory that connects the two semantical approaches mentioned above has potential for shedding significant light on completeness phenomena, as well as being of intrinsic interest.

The paper under review not only gives a quite readable and leisurely introduction to the important issues of the subject, but delineates a new connection between the algebraic and relational approaches, said connection taking the form of an adjoint pair of functors between certain categories. (This adjunction extends known duality theorems that arise from an analysis of the classical duality of M. H. Stone. This is where topology enters the picture.)

The paper is divided into three chapters. Chapter I introduces the reader to modal semantics, discusses the ideas upon which duality theory is based, and points out the relevance of these ideas to completeness theorems. Chapter II develops the technical machinery leading to the adjoint functor theorem alluded to above. Finally, Chapter III gives applications of the theory developed in Chapter II; one such is a simple description of the structure of modal classes of "generalized" Kripke frames.

Reviewer: [P.Bankston](#)

MSC:

03B45 Modal logic (including the logic of norms)

18A40 Adjoint functors (universal constructions, reflective subcategories, Kan extensions, etc.)

Cited in **1** Review
Cited in **29** Documents

Keywords:

[general frames](#); [valuations](#); [duality](#); [adjoint functors](#); [semantics of propositional modal logics](#); [modal algebras](#); [adjoint pair of functors](#); [completeness theorems](#); [Kripke frames](#)

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