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Chierchia, Luigi

Absolutely continuous spectra of quasiperiodic Schrödinger operators. (English)

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This paper is concerned with the spectral theory of one-dimensional Schrödinger operators $L \equiv -d^2/dx^2 + v(x)$, where v is a (real) quasiperiodic function. The eigenvalue problem is discussed both from an abstract and a constructive point of view. For example, a general formula for the absolutely continuous (a.c.) spectral densities that yields an immediate proof of the fact that the Kolmogorov-Arnold-Moser (KAM) spectrum constructed by *E. I. Dinaburg*, *Ja. G. Sinai* [Funkt. Anal. Prilozen. 9, 8-21 (1975; Zbl 0333.34014) and *H. Rüssmann*, Nonlinear dynamics, int. Conf., New York 1979, Ann., N. Y. Acad. Sci. 357, 90-107 (1980; Zbl 0477.34007)] is a subset of the a.c. spectrum, is derived. Also, it is shown that the a.c. (generalized) eigenfunctions are "weak" Bloch waves, generalizing, in this sense, Floquet theory to the a.c. part of the spectrum of L. The problem of constructing explicitely smooth Bloch waves is then considered and the Dinaburg-Sinai-Rüssmann theory is extended to quasiperiodic perturbations of periodic Schrödinger operators. The existence of such Bloch waves is shown to be intimately related to the canonical integrability of $a(d+1)$-dimensional (d≡ # of basic frequencies of v) classical Hamiltonian system parametrized by the eigenvalue E. Particular attention is devoted to the dependence upon E and a complete control of KAM objects is achieved using the notion of Whitney smoothness [*H. Whitney*, Trans. Am. Math. Soc. 36, 63-89 (1934; Zbl 0008.24902)].

Reviewer: L.Chierchia

MSC:

34L99	Ordinary differential operators
35J10	Schrödinger operator, Schrödinger equation
37C55	Periodic and quasi-periodic flows and diffeomorphisms
34K99	Functional-differential equations (including equations with delayed, advanced or state-dependent argument)
47A10	Spectrum, resolvent

Keywords:

spectral theory of one-dimensional Schrödinger operators; spectral densities; Bloch waves; KAM objects

Full Text: DOI

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