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Limit theorems for stochastic processes. (English) Zbl 0635.60021

Grundlehren der Mathematischen Wissenschaften, 288. Berlin etc.: Springer-Verlag. XVIII, 601 p.; DM 198.00 (1987).

This monograph is an excellent self-contained presentation of the theory of convergence in law for processes. The class of stochastic processes which is considered is that of semimartingales which is broad enough to cover most common stochastic processes as time discrete processes, most Markov processes, solutions of stochastic differential equations or point processes.

The stochastic calculus or general theory of stochastic processes is the powerful tool which allows the deep study of these processes. The first two chapters give a rather complete account of this theory. There exist several books that give a detailed presentation of the general theory of stochastic processes, but the authors restricted themselves in a successful manner to the needed facts for the later investigation of weak convergence of semimartingales. These first two chapters represent more a survey than a beginner's course and refresh the readers mind about notation and definitions. They contain sections on semimartingales, stochastic integrals, random measures, characteristics of semimartingales and processes with independent increments.

Chapter III introduces the so-called martingale problems which are related to the question: what are all the probability measures on a given filtered probability space under which all members of a given family of processes are local martingales? Attention is also payed to the relation between martingale problems and characteristics of semimartingales. Further, this chapter considers changes of measures and gives various versions of Girsanov's theorem.

Chapter IV studies the question of absolute continuity or singularity of two probability measures which are laws of semimartingales. Here the so called Hellinger processes play a crucial role. In Chapter V the Hellinger processes are applied to the problem of continuity and entire separation for two sequences of measures. Also convergence in variation is considered.

The Skorokhod topology and basic facts about weak convergence of measures and convergence in law can be found in Chapter VI. Several tightness criteria are developed especially suited to semimartingales.

The Chapters VII-IX represent the heart of the book. Chapter VII studies the convergence of processes with independent increments. It represents the first step in formulating functional limit theorems for sequences of semi-martingales toward a semimartingale. In Chapter VIII functional limit theorems are proved where only the limiting process has independent increments. Finally in the limit theorems of Chapter IX also the limiting process itself belongs to some rather broad class of semimartingales.

Many authors have proved various versions of functional limit theorems which concern weak convergence of a sequence of semimartingales toward a semimartingale. This monograph gives at the first time a rather complete presentation of this modern field of research. The underlying most powerful methods are mainly based on convergence of martingales and on the relations between semimartingales and their characteristics.

The last chapter is statistically motivated and treats contiguity and convergence of processes. Especially the convergence of likelihood ratio processes to a process with independent increments and the statistical invariance principle are considered. At the end of the book the reader finds bibliographical comments, an extensive bibliography and a helpful index of symbols and terminology.

The authors succeeded in writing a rather complete book about the modern theory of limit theorems for semimartingales which will become a standard reference and may play an important role in further applications and developements of this theory.

Reviewer: [E.Platen](#)

MSC:

- 60Fxx Limit theorems in probability theory
- 60-02 Research exposition (monographs, survey articles) pertaining to probability theory
- 60B10 Convergence of probability measures
- 60H05 Stochastic integrals
- 60G44 Martingales with continuous parameter
- 60G48 Generalizations of martingales

Cited in **19** Reviews
Cited in **631** Documents

Keywords:

theory of convergence in law for processes; semimartingales; weak convergence of semimartingales; processes with independent increments; martingale problems; Hellinger processes; convergence in variation; Skorokhod topology; tightness criteria; functional limit theorems; convergence of likelihood ratio processes