Sinha, B. K.; **Ghosh, M.**

Inadmissibility of the best equivariant estimators of the variance- covariance matrix, the precision matrix, and the generalized variance under entropy loss. (English) Zbl 0634.62050

Stat. Decis. 5, No. 1-4, 201-227 (1987).

Let $X = [X_1, ..., X_k]$ (p×k) have independent columns, with $X_i \sim N(\xi_i, \Sigma)$ and the ξ_i and Σ (p×p nonsingular) are unknown. Let S (p×p) $\sim W(n, \Sigma)$ be a Wishart matrix independent of X. Estimation of Σ by $\hat{\Sigma}$ is studied under two loss functions:

$$L_1(\hat{\Sigma}, \Sigma) = tr(\hat{\Sigma}\Sigma^{-1}) - \log|\hat{\Sigma}\Sigma^{-1}| - p, \quad and \quad L_2(\hat{\Sigma}, \Sigma) = tr(\Sigma\hat{\Sigma}^{-1}) - \log|\Sigma\hat{\Sigma}^{-1}| - p.$$

Under L_1 the best equivariant estimator is $n^{-1}S$. An improved estimator (called "testimator", based on an idea of C. Stein) is presented, defined by: $\hat{\Sigma} = n^{-1}S$ unless Roy's maximum root test accepts the hypothesis that all ξ_i are 0 in which case $\hat{\Sigma} = (n+k)^{-1}(S + XX')$. A similar testimator is found under L_2.

The results are used for simultaneous estimation of μ nd Σ in a sample from a p-variate N(μ,Σ) population. Estimation of $|\Sigma|$ is handled in a similar way. Monte Carlo results on the improvement of the estimators are presented. Some results on sequential estimation are also given.

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MSC:

62H12	Estimation in multivariate analysis
62C15	Admissibility in statistical decision theory
62F10	Point estimation
62L12	Sequential estimation

Keywords:
inadmissibility; variance-covariance matrix; multinormal population; entropy loss; generalized variance; MANOVA test; Wishart matrix; best equivariant estimator; testimator; Roy's maximum root test; Monte Carlo results