Freidlin, M. I.

Geometric optics approach to reaction-diffusion equations. (English) Zbl 0626.35047

SIAM J. Appl. Math. 46, 222-232 (1986).

The limit $\lim_{\epsilon\searrow 0} u^\epsilon(x,t)$ is studied, where $u^\epsilon(x,t)$ is the solution of

$$\partial u^\epsilon(t,x)/\partial t=\epsilon/2\sum_{i,j=1}^{r}\partial/\partial x^i(a^{ij}(x)\partial u^\epsilon/\partial x^j)+(1/\epsilon)f(x,u^\epsilon);\quad u^\epsilon(x,0)=g(x)\geq 0,$$

for $t>0$ and $x\in\mathbb{R}^r$. It is shown that under appropriate hypotheses the solution tends to a function which only takes the values 0 or 1. The evolution of the limit function with time amounts to the change of the set where this function is equal to 1. It is shown that for f belonging to a suitable class of functions this set changes in accordance with Huygens principle for an appropriate velocity field which can be expressed in terms of the diffusion coefficients and the nonlinear term. Also other classes for the function f are studied where more complicated phenomena appear. Moreover, the case of systems is also shortly discussed.

Reviewer: H.D.Alber

MSC:

35K55 Nonlinear parabolic equations
35B25 Singular perturbations in context of PDEs
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

propagation of traveling waves; evolution of the limit function; Huygens principle; velocity field; diffusion coefficients

Full Text: DOI