Koganov, L. M.

Combinatorial proof of Deddens' theorem. (Russian) Zbl 0615.10004

Algebraic systems with one operation and one relation, Interuniv. Collect. sci. Works, Leningrad 1985, 46-53 (1985).

[For the entire collection see Zbl 0592.00016.]

Let m and n be relatively prime positive integers. Put $G = \{um + vn$; u and v are arbitrary non-negative integers$\}$. For any $g \in G$ we denote by N(j,g) the number of ways of representing g as the sum of j non-zero elements of G. Put $L(0) = 1$ and $L(g) = \sum_{j=1}^{\infty}(-1)^j N(j,g)$ for $g \in G$, $g \neq 0$.

The author gives a combinatorial proof of the following theorem of *J. A. Deddens* [J. Comb. Theory, Ser. A 26, 189-192 (1979; Zbl 0414.05005)]: If $g \equiv 0$ or $m + n$ (mod mn), then $L(g) = 1$. If $g \equiv m$ or n (mod mn), then $L(g) = -1$. Otherwise $L(g) = 0$.

Reviewer: B.Pondeliček

MSC:

11A07	Congruences; primitive roots; residue systems
11A25	Arithmetic functions; related numbers; inversion formulas
06F05	Ordered semigroups and monoids

Keywords:

congruences; inversion formulas; ordered semigroups