Manz, Olaf; **Staszewski, Reiner**

On the number of generators and the modular group-ring of a finite p-group. (English)
Zbl 0611.20003

Proc. Am. Math. Soc. 98, 189-195 (1986).

Let K be a field of characteristic $p > 0$ and let P be a finite p-group. Let $JK(P)$ be the Jacobson radical of the group ring $K(P)$. A particular descending central series, the Loewy series, is defined by letting $\kappa_1(P) = P$, $\kappa_n(P) = [\kappa_{n-1}(P), P]\kappa_m(P)^p$ where m is the least integer for which $pm \geq n$ $(n = 1, 2, \ldots, \ell + 1)$ where $\kappa_\ell(P) \neq 1$, $\kappa_{\ell+1}(P) = 1$. Let $p^{d_n} = |\kappa_n(P) : \kappa_{n+1}(P)|$ $(n = 1, 2, \ldots, \ell)$ and $c_i = \dim_K JK(P)^i/JK(P)^{i+1}$ $(i = 0, 1, ..., s)$ where $JK(P)^s \neq 0$, $JK(P)^{s+1} = 0$. By the work of *S. A. Jennings*

$$\prod_{n=1}^{\ell}(1 + t^n + ... + t^{n(p-1)})^n = \sum_{i=0}^{s} c_i t^i$$

[Trans. Am. Math. Soc. 50, 175-185 (1941; Zbl 0025.24401)]. The Loewy series is said to be "even monotonic" if $c_{i-1} \leq c_i$ $(1\leq i \leq s/2)$. It is shown, by example, that the Loewy series is not always even monotonic but that it is so if P is either Abelian, p-regular or extra-special. Noting that $d = d_1 = c_1$ is the minimal number of generators of P it is shown that $c_n \geq d$ $(1\leq n \leq s - 1)$, generalizing a result of *B. Külshammer* [J. Algebra 88, 190-195 (1984; Zbl 0567.20007)]. Other results of interest on the Loewy series are given.

Reviewer: D.A.R.Wallace

MSC:

20C05	Group rings of finite groups and their modules (group-theoretic aspects)
20D15	Finite nilpotent groups, p-groups
20C20	Modular representations and characters
16S34	Group rings
16Nxx	Radicals and radical properties of associative rings
20F05	Generators, relations, and presentations of groups

Keywords:

finite p-groups; Jacobson radical; group rings; descending central series; Loewy series; minimal number of generators

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