

Amick, C. J.; Fraenkel, L. E.

The uniqueness of Hill's spherical vortex. (English) Zbl 0609.76018
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The authors study the free boundary problem

$$r\left(\frac{1}{r}\psi_r\right)_r + \psi_{zz} = \begin{cases} -\lambda r^2 f_0(\psi) & \text{in } A; \\ 0 & \text{in } \Pi \setminus A, \end{cases}$$

$\psi|_{r=0} = -k$, $|\partial_A = 0$ together with certain asymptotics at infinity.

Here $\Pi = \{(r, z) \mid r > 0, z \in \mathbb{R}\}$, $f_0 \geq 0$, and ψ is a Stokes stream function in cylindrical co-ordinates (no dependence on θ). The set $A \subset \Pi$ is bounded and open, but a priori unknown. A special case of the problem is Hill's problem, in which an explicit solution is known. It is proven that any weak solution to the problem is the explicit solution modulo a translation in z . Such solutions may be obtained as local maximizers of functional.

Reviewer: [G. Warnecke](#)

MSC:

[76B47](#) Vortex flows for incompressible inviscid fluids
[35J25](#) Boundary value problems for second-order elliptic equations
[35R35](#) Free boundary problems for PDEs

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Keywords:

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