Mohammed, S.-E. A.

Stochastic functional differential equations. (English) Zbl 0584.60066
Research Notes in Mathematics, 99. Boston-London-Melbourne: Pitman Advanced Publishing Program. VI, 245 p. £12.95 (1984).

This research note provides an account of qualitative theory of stochastic functional differential equations (SFDE). The book has 7 chapters as follows: I. Preliminary background; where the author gives some results from probability theory and linear analysis necessary in the volume; II. Existence of solutions and dependence on the initial process; where is established existence, uniqueness and continuous dependence on the initial process for solutions to general SFDE's of the form:

$$x(\omega)(t) = \begin{cases} \theta(\omega)(t), & -r \le t \le 0; \\ \theta(\omega)(0) + (\omega)\int_0^t g(u, x_u)dz(.)(u), & 0 \le t \le a; \end{cases}$$

where x_u is defined by $x_u(\omega)(s) = x(\omega)(u+s)$, $\omega \in \Omega$, $s \in [-r, 0]$, and the stochastic integral is a McShane belated integral.

In chapter III - Markov trajectories - it is shown that the trajectory field $\{\eta_{x_t}, t \ge 0\}$ defined by the equation

$$dx(t) = H(t, x_t)dt + G(t, x_t)dw(t), \quad t \ge 0,$$

$$x(\omega)(t) = \eta(t), \quad t \in [-r, 0]; \quad \eta \in C = C([-r, 0]; R^n)$$

is a Feller process on the state space C. If the SFDE is autonomous then the trajectory field is a time-homogeneous diffusion on C. In chapter IV - The infinitesimal generator - the author considers the autonomous SFDE

$$dx(t) = H(x_t)dt + G(x_t)dw(t)$$

and investigates the structure of the associated one-parameter semi-group $(P_t)_{t\ge0}$ given by the time-homogeneous diffusion on C; it is proved that the semi-group $(P_t)_{t\ge0}$ is never strongly continuous on the Banach space $C_b = C_b(C; R)$ of all bounded uniformly continuous functions $\Phi : C \to R$; the weak generator A of $(P_t)_{t\ge0}$ is defined and a concret form of A on a subspace of C_b is found.

In chapter V - Regularity of the trajectory field - there are explored distributional and sample regularity properties for trajectory fields of autonomous SFDE's. Chapters VI and VII contain examples of SFDE (stochastic delay equations, linear functional differential equations forced by white noise, a model for physical Brownian motion, stochastic integro-differential equations, SFDE's with an infinite memory) and some open problems.

Reviewer: R.Aurel

MSC:

60Hxx	Stochastic analysis
60-02	Research exposition (monographs, survey articles) pertaining to probability theory
60J60	Diffusion processes
60G17	Sample path properties

Keywords:

stochastic functional differential equations; McShane belated integral; time-homogeneous diffusion; sample regularity properties; stochastic delay equations; stochastic integro-differential equations