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Theory of multiobjective optimization. (English) Zbl 0566.90053

Mathematics in Science and Engineering, Vol. 176. Orlando etc.: Academic Press, Inc. (Harcourt Brace Jovanovich, Publishers). XIII, 296 p. $ 48.00; £42.00 (1985).

Multiobjective optimization, i.e. dealing with situations when several objective functions must be optimized, is usually considered as an application field for heuristic methods. The book under review is perhaps the first monograph where this topic is described as a unified mathematical discipline with deep general theory. After some mathematical preliminaries (convex sets, etc.) different notions of solutions are introduced and investigated; the best known are Pareto optimality, Neumann-Morgenstern solution, kernel etc. Ch. 4 is devoted to the problems of stability for perturbation of initial data (including stability for perturbation of the dominance structure). Ch. 5 and 6 discuss generalizations of the Lagrange multiplier method and the notion of duality (usual in linear and convex programming) to multiobjective optimization. Ch. 7 is concerned with methodology: the notion of utility function is introduced, some methods of using it are given, and computer- ready algorithms are given in ch. 7.3.

The main drawbacks: the exposition is mathematically very interesting, but almost no motivation appears until ch. 7, and there is still a serious gap between general theorems and heuristic methods within ch. 7, that either reduce the problem $c_i \to \min$ to $f(c_1,...,c_n) \to \min$ for some function f or to some man-machine interactive process. Several methods are completely neglected in the book, among them the symmetry approach (the most perspective method to the reviewer's viewpoint). Sometimes we do not know the total objective function, but we know that the dominance relation between outputs is symmetric. Therefore if we choose some unique solution, it must be symmetric with respect to the same symmetry group. E.g. for group decision invariance with respect to an arbitrary permutation of participants and to arbitrary rescaling of their utility functions $u_i \to c_i u_i$ leads to Nash's solution $u_i \to \max$. This approach is very fruitful, e.g. in case we choose between arbitrary lotteries of alternatives (this is a usual trick in game theory - to give a lottery as a solution, where the lottery has actions as outcomes), Nash's solution solves Arrow's paradox ("impossibility of group decision") that the authors claim to be still unsolved (see, e.g., the reviewer's abstract in Notices AMS 25, No.7, A- 703 (1978)). These drawbacks, however, are excusable and do not prevent the book from being a brilliant mathematical survey of the subject.

Reviewer: V.Ya.Kreĭnovich

MSC:

90C31	Sensitivity, stability, parametric optimization
90B50	Management decision making, including multiple objectives
91A80	Applications of game theory
91B16	Utility theory
49-02	Research exposition (monographs, survey articles) pertaining to calculus of variations and optimal control
90-02	Research exposition (monographs, survey articles) pertaining to operations research and mathematical programming
90C90	Applications of mathematical programming
91B08	Individual preferences
91B10	Group preferences

Keywords:

Lagrange duality; conjugate duality; point-to-set maps; vector optimization; Multiobjective optimization; Pareto optimality; Neumann- Morgenstern solution; kernel; stability; perturbation of initial data; perturbation of the dominance structure; Lagrange multiplier method