Zucker, Steven

Satake compactifications. (English) Zbl 0565.22009

Comment. Math. Helv. 58, 312-343 (1983).

In a fundamental paper [Comment. Math. Helv. 48, 436–491 (1973; Zbl 0274.22011)] *A. Borel* and *J. P. Serre* constructed a compactification of the locally symmetric space $\Gamma \setminus X$; where Γ is an arithmetic subgroup of the group G of isometries of a symmetric space X with nonpositive sectional curvatures.

The main result of the paper under review is that the Satake compactification of $\Gamma \setminus X$ for certain representations $\bullet$ (for example if τ is defined over Q) is a quotient of the Borel-Serre compactification. Inspired by the construction of Borel and Serre of the corner $X(P)$ associated with a parabolic Q-subgroup P, given a finite dimensional representation $\bullet$ of G, the author constructs the "crumpled corner" $X^*(P)$ which is a quotient of $X(P)$. For parabolic Q-subgroups $P \subset Q$, $X^*(Q)$ is embedded in $X^*(P)$ as an open subset and this embedding is compatible with the projections $X(P) \to X^*(P)$ and $X(Q) \to X^*(Q)$. Let ${}_Q\tilde{X}^*$ be the union of the $X^*(P)'s$. Then ${}_Q\tilde{X}^*$ is a quotient of the manifold $\bar{X}$ with corners. The construction of the crumpled corner is such that it is seen at once that there is a natural bijection of ${}_Q\tilde{X}^*$ onto ${}_QX^*_\tau$. It is proved here that this bijection is continuous. Now it follows immediately that the Satake compactification $\Gamma \setminus_Q X^*_\tau$ is a quotient of the Borel-Serre compactification $\Gamma \setminus \bar{X}$.

If X is Hermitian *W. L. Baily* jun. and *A. Borel* [Ann. Math. (2) 84, 442–528 (1966; Zbl 0154.08602)] gave a compactification of $\Gamma \setminus X$ which is a normal analytic space. It is shown in this paper that the Baily-Borel compactification is homeomorphic to the Satake compactification with respect to any representation of G whose restricted highest weight is a multiple of the distinguished fundamental dominant weight. As a consequence, it follows that the Baily-Borel compactification is also a quotient of the Borel-Serre compactification.

Reviewer: Gopal Prasad (Bombay)

MSC:

22E40 Discrete subgroups of Lie groups
20G20 Linear algebraic groups over the reals, the complexes, the quaternions
32J05 Compactification of analytic spaces

Keywords:

compactification; locally symmetric space; Satake compactification; Borel-Serre compactification; crumpled corner; Baily-Borel compactification

Full Text: DOI EuDML