**Deift, P.**; **Simon, B.**

**Almost periodic Schrödinger operators. III: The absolutely continuous spectrum in one dimension.** (English) Zbl 0562.35026

Commun. Math. Phys. 90, 389-411 (1983).

Summary: [For parts I and II see *J. Avron* and the second author, ibid. 82, 101-120 (1981; Zbl 0484.35069) and Duke Math. J. 50, 369-391 (1983; Zbl 0544.35030) respectively.]

We discuss the absolutely continuous spectrum of $H=-d^2/dx^2+V(x)$ with V almost periodic and its discrete analog $(hu)(n)=u(n+1)+u(n-1)+V(n)u(n)$. Especial attention is paid to the set, A, of energies where the Lyaponov exponent vanishes. This set is known to be the essential support of the a.c. part of the spectral measure.

We prove for a.e. V in the hull and a.e. E in A, H and h have continuum eigenfunctions, u, with $|u|$ almost periodic. In the discrete case, we prove that $|A|\le 4$ with equality only if $V=const$. If k is the integrated density of states, we prove that on A, $2kdk/dE\ge \pi^{-2}$ in the continuum case and that $2\pi\sin\pi kdk/dE\ge 1$ in the discrete case. We also provide a new proof of the Pastur-Ishii theorem and that the multiplicity of the absolutely continuous spectrum is 2.

**MSC:**

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|---|---|
| 35J10 | Schrödinger operator, Schrödinger equation |
| 35P05 | General topics in linear spectral theory for PDEs |
| 35B15 | Almost and pseudo-almost periodic solutions to PDEs |



**Keywords:**

absolutely continuous spectrum; essential support; continuum eigenfunctions; Pastur-Ishii theorem; multiplicity of the absolutely continuous spectrum

**Full Text:** DOI

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