Ehresmann, Charles

Oeuvres complètes et commentées. Topologie algébrique et géométrie différentielle. Parties I-1 et I-2. Éd. par Andrée Charles Ehresmann. (French) Zbl 0561.01027

Suppléments 1 et 2 au Vol. XXIV (1983) des Cahiers de Topologie et Géométrie Différentielle. Amiens: A. C. Ehresmann, U.E.R. de Mathématiques. XXIV, 5, 601 p. (1984).

This is the first part of the collected works of Charles Ehresmann, although chronologically it is the last to be published. The format is different from the other volumes in a number of ways. First, both sections appear in the same volume and the table of contents makes no distinction between I-1 and I-2. Thus altogether there are seven volumes rather than eight. (See the following review for the location of the reviews of the other six volumes.) Second, the contents of this volume, representing Ehresmann's work in topology and differential geometry, involve almost no category theory. All papers in the general list of papers with numbers greater than or equal to 50 are about category theory, but four of them are reprinted here since they are directly concerned with geometry. Third, the Comments are not written by A. C. Ehresmann as in the other volumes, but instead are general review articles by 12 different authors about various aspects of Ehresmann's work in geometry, topology and category theory. We list them here since these papers do not appear in the general table of contents but rather in a separate table of contents at the beginning of the Comments section.

I. The papers of Charles Ehresmann on homogeneous spaces and Lie groups, by *W. T. Van Est*; II. Les travaux de Charles Ehresmann sur les espaces fibres, by *Fm. Zisman*; III. Variétés feuilletées, by *G. Reeb*; IV. Les travaux de Charles Ehresmann en géométrie différentielle, by *P. Libermann*; V. La théorie des jets et ses développements ultérieurs, by *R. Thom*; VI. Au coeur de l'oeuvre de Charles Ehresmann et de la géométrie différentielle: Les groupoides différentiables, by *J. Pradines*; VII. The work of Charles Ehresmann in the 1950's and its applications in physics and control theory, by *R. Hermann*; VIII. Ehresmann and the fundamental structures of differential geometry seen from a synthetic viewpoint, by *A. Kock*; IX. Ehresmann: un géomètre, by *A. Haefliger*; X. Sur les structures locales de C. Ehresmann, by *J. Benabou*; XI. Sur les contributions de Charles Ehresmann à la théorie des catégories, by *R. Guitart*; XII. Charles Ehresmann, by *A. C. Ehresmann*; XIII. Liste des thèses préparées sous la direction de Charles Ehresmann.

Doubtless these papers will not be reviewed on their own, but that is all right since A. C. Ehresmann has reviewed them in lieu of the usual Synopsis section. All in all, this is a curiously convoluted and self- referential work. As A. C. Ehresmann points out, C. Ehresmann's work contains about 500 pages of topology and differential geometry (up to 1959), followed by 1800 pages of category theory. These first 500 pages are a firm foundation for Ehresmann's future reputation, containing his well-known works on homogeneous spaces, fibre spaces, foliations, connections, local structures, jets, holonomy, prolongations, etc. Ehresmann himself wrote a review of this material in 1955 when he was a candidate for a position at the University of Paris. Is appears in this volume as paper 136.

Why did Ehresmann abandon this extremely fruitful line of research, which is still being worked on today, and turn his attention in 1959 completely to category theory? We will probably never know exactly how he expected category theory to lead to new results in geometry. His few brief papers about categories and differential geometry consist mainly of definitions together with generalized constructions that mimic familiar constructions in differential geometry without leading to any new results or tools for geometry. One can't help thinking that if someone with Ehresmann's geometrical insight was unable to wring new geometrical results from category theory, then what chance is there for the rest of us to do so. (The answer of course is that we will use much more powerful categorical tools.) Several of the papers listed above address this question. The most helpful are those by Pradines and A. C. Ehresmann. What Kock says about synthetic differential geometry being a continuation of Ehresmann's work is surely true, but there is no evidence that he himself anticipated any such development. Benabou rightly emphasizes Ehresmann's work on locales, which was ignored at the time it was done, but I don't see how Barr's theorem about the existence of sufficiently many locales makes Ehresmann's work a predecessor of topos theory. One could better argue for the work on multiple categories being a precursor of current categorical work in homotopy theory. I myself suspect that while Ehresmann may or may not have expected or intended any great geometrical enlightenment to flow from his work on categories, he surely went where his curiosity and current interest took him. Since he was one of the notable mathematicians of our century, the results were bound to be interesting.

Reviewer: J.W.Gray

MSC:

01A75	Collected or selected works; reprintings or translations of classics
55-03	History of algebraic topology
53-03	History of differential geometry

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differential geometry; topology; category theory